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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

October 2024 Supplementary Examinations

Programme: B.E. **Semester: III**
Branch: Machine Learning **Duration: 3 hrs.**
Course Code: 23MA3BSMML **Max Marks: 100**
Course: Mathematical Foundation for Machine Learning - 1

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	*CO*	*PO*	**Marks**
1	a)	Check whether the linear transformation $T:\mathbb{R}^5 \to \mathbb{R}^4$ defined by $T = \begin{bmatrix} 2 & -1 & 1 & -6 & 8 \\ 1 & -2 & -4 & 3 & -2 \\ -7 & 8 & 10 & 3 & -10 \\ 4 & -5 & -7 & 0 & 4 \end{bmatrix}$ is one-one and onto. If not one-one, find the non-zero vector whose image is a zero vector.	*CO1*	*PO1*	**06**
	b)	Let $G: R^3 \to R^3$ be linear transformation defined by $G(x, y, z) = (2x, 4x-y, 2x+3y-z)$ (i) verify whether G is non-singular or not. (ii) Find G^{-1} if it exists. Also find a preimage of (4, 7, 7).	*CO1*	*PO1*	**07**
	c)	Apply Gaussian elimination approach to find the determinant of the matrix $A = \begin{bmatrix} 4 & -1 & 2 & 1 & 0 \\ 1 & 2 & 1 & 5 & 4 \\ 2 & -1 & 3 & 1 & 3 \\ 3 & 1 & 6 & 3 & 1 \\ 7 & 2 & 8 & 9 & 3 \end{bmatrix}$. Hence write the number of additions and multiplications you have used to get the result.	*CO1*	*PO1*	**07**
		UNIT - 2			
2	a)	Let $G: R^3 \to R^3$ be a linear transformation defined by $G(x, y, z) = (2x+y, x-y+z, 3y-z)$. (i) Find G^2, G^3. (ii) Also find $G^2(2,1,3)$ and $G^{-2}(2,1,3)$.	*CO1*	*PO1*	**06**
	b)	Find the image of the triangle having vertices $A(1,3,1)$, $B(5,3,1)$ and $C(4,6,1)$ in homogeneous coordinates under the sequence of transformations T followed by R followed by S where $T = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$, $R = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $S = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$. Sketch the original and the final triangle.	*CO1*	*PO1*	**07**

		c)	Given $G: R^2 \to R^2$ and $S = \{u_1, u_2\} = \{(1, 3), (2, 5)\}$ is the basis of R^2 and $G(x, y) = (2x - 7y, 4x + 3y)$, (i) Find the matrix of G_s, relative to S. (ii) Verify $[G]_s . [v]_s = [G(v)]_s$ for the vector $v = (4, -3)$ in R^2.	CO1	PO1	**07**
			UNIT - 3			
	3	a)	Design and implement a pseudo code to find the condition number of the matrix $B = \begin{bmatrix} 10 & 2 \\ 7 & -5 \end{bmatrix}$ using Frobenious norm.	CO2	PO5	**06**
		b)	Prove that vector space $V = \mathbb{R}^3$ be an inner product space vector space with respect to inner product $\langle u, v \rangle = u_1 v_1 - u_2 v_1 - u_1 v_2 + 4u_2 v_2$. Is this a normed vector space?	CO1	PO1	**07**
		c)	Show that $S = \{u_1, u_2, u_3, u_4\}$, where $u_1 = (1,1,0,-1)$, $u_2 = (1,2,1,3), u_3 = (1,1,-9,2), u_4 = (16,-13,1,3)$ is an orthogonal basis of $\mathbb{R}^4$. Find the coordinates of the arbitrary vector $v = (a,b,c,d)$ in $\mathbb{R}^4$ relative to the basis S.	CO1	PO1	**07**
			OR			
	4	a)	Find the value of α such that the matrices $A = \begin{bmatrix} \alpha & 8 & -7 \\ 6 & 5\alpha & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 5 & 6\alpha \end{bmatrix}$ are orthogonal with respect to an inner product $\langle A, B \rangle = Tr(B^T A)$. Hence find $\Vert A \Vert$ and $\Vert B \Vert$.	CO1	PO1	**06**
		b)	Let u and v are any two vectors in an inner product space V. (i) If $\Vert u \Vert = \Vert v \Vert$ then prove that $\langle u+v, u-v \rangle = 0$, (ii) Expand $\langle -5u + 3v,\ -4u + 2v \rangle$.	CO1	PO1	**07**
		c)	For the functions $f(t) = t^2 - 1$ and $g(t) = 3t$ in polynomial space $P(t)$ verify the following: (i) $\Vert f + g \Vert \leq \Vert f \Vert + \Vert g \Vert$ (ii) $\Vert f + g \Vert^2 + \Vert f - g \Vert^2 = 2(\Vert f \Vert^2 + \Vert g \Vert^2)$, given $\langle f, g \rangle = \int_0^1 fg\,dt$.	CO1	PO1	**07**
			UNIT – 4			
	5	a)	Let $d(u, v)$ be the metric in the vector space R^n. Verify whether $d^*(u, v) = \frac{d(u,v)}{1 + d(u,v)}$ is a metric on R^n.	CO1	PO1	**06**
		b)	Construct an orthogonal basis and hence an orthonormal basis of the subspace W spanned by the vectors $v_1 = (1,1,1,1)$, $v_2 = (1,2,4,5)$ and $v_3 = (1,-3,-4,-2)$ of $\mathbb{R}^4$ using Gram-Schmidt orthogonalization process.	CO1	PO1	**07**
		c)	A sales organization obtains the following data relating the number of salespersons to annual sales. x: Number of salespersons: 5, 6, 7, 8, 9, 10 y: Annual Sales (million dollars): 2.3, 3.2, 4.1, 5.0, 6.1, 7.2 Find the least squares line of the form $y = a + bx$ and hence estimate the annual sales when there are 14 salespersons.	CO1	PO1	**07**

x: Number of salespersons	5	6	7	8	9	10
y:Annual Sales (million dollars)	2.3	3.2	4.1	5.0	6.1	7.2

			UNIT - 5			
	6	a)	Design and implement the algorithm to find the spectral norm of a matrix $A = \begin{bmatrix} 2 & -5 \\ -2 & 3 \\ 1 & 7 \end{bmatrix}$.	*CO1*	*PO1*	**06**
		b)	Find the eigenspaces of the linear transformation $T: P_2(t) \rightarrow P_2(t)$ defined by $T(f(t)) = (a + 3b + 3c)t^2 + (-3a - 5b - 3c)t + (3a + 3b + c)$, where $f(t) = at^2 + bt + c$	*CO2*	*PO5*	**07**
		c)	Check whether the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ is diagonalizable. If so, find the modal matrix which diagonalize A.	*CO1*	*PO1*	**07**
			OR			
	7	a)	Given $A = \begin{bmatrix} 3 & 1 \\ 1 & -3 \end{bmatrix}$ compute A^9 without using the conventional matrix multiplication. Show all the necessary computations.	*CO1*	*PO1*	**06**
		b)	Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, orthogonally diagonalize $A^T A$.	*CO1*	*PO1*	**07**
		c)	Find the characteristic and minimal polynomial of the matrix $A = \begin{bmatrix} 2 & 5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 3 & 5 & 0 \\ 0 & 0 & 0 & 0 & 7 \end{bmatrix}$.	*CO1*	*PO1*	**07**

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