U.S.N.										

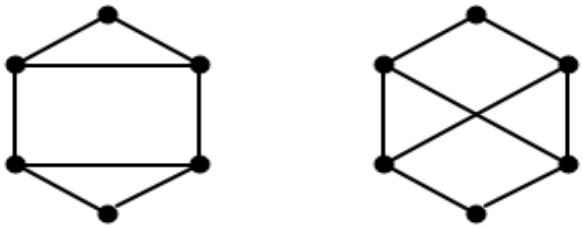
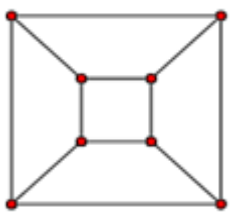
B.M.S. College of Engineering, Bengaluru-560019

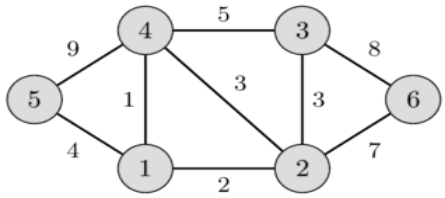
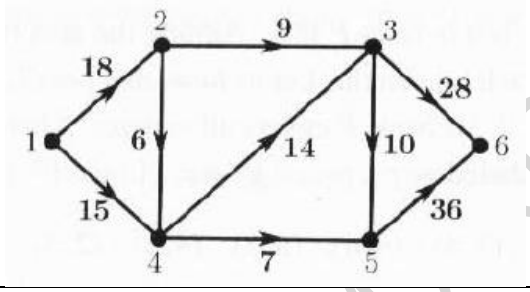
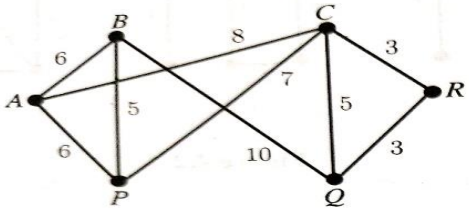
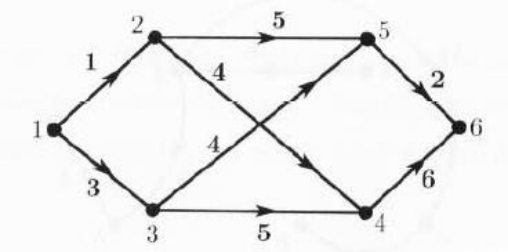
Autonomous Institute Affiliated to VTU

June 2025 Semester End Main Examinations

Program : **B. E** **Semester** : **III**
Branch : **AI and ML** **Duration** : **3 Hrs.**
Course Code : **22MA3BSMML** **Max. Marks** : **100**
Course : **Mathematical Foundations for Machine Learning**
Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

			CO	PO	Marks
		UNIT - I			
1	a)	Solve the linear congruence $7x \equiv 2 (\text{mod } 37)$.	1	1	**6**
	b)	Apply the Chinese remainder theorem, solve the linear congruences $x \equiv 2 (\text{mod } 3)$, $x \equiv 3 (\text{mod } 5)$, and $x \equiv 2 (\text{mod } 7)$.	1	1	**7**
	c)	Generate the public and private keys to encrypt certain messages using primes 3 and 11.	2	1	**7**
		OR			
2	a)	Solve the linear congruence $13x \equiv 10 (\text{mod } 28)$.	1	1	**6**
	b)	Solve the polynomial congruence $x^3 + x + 3 \equiv 0 \text{ (mod } 25)$.	1	1	**7**
	c)	Apply Fermat's little theorem to find the reminder when 24^{1947} is divided by 17.	1	1	**7**
		UNIT - II			
3	a)	Define simple graph, regular graph and induced subgraphs with examples.	1	1	**6**
	b)	Examine whether the graphs given below are isomorphic or not? 	1	1	**7**
	c)	Prove that a connected graph G has a Euler circuit if and only if all vertices of G are of even degree.	1	1	**7**
		OR			
4	a)	Define Euler trail, Euler circuit, Hamilton path and Hamilton cycle.	1	1	**6**
	b)	Determine the adjacency and the incidence matrix of the graph given below 	1	1	**7**
	c)	Prove that a connected graph with n vertices has at least $(n-1)$ edges.	1	1	**7**

			UNIT - III			
	5	a)	Let $T_1 = (V_1, E_1)$ and $T_2 = (V_2, E_2)$ be two trees with $\lvert E_1 \rvert = 19$ and $\lvert V_2 \rvert = 3\lvert V_1 \rvert$, then determine $\lvert V_1 \rvert$, $\lvert V_2 \rvert$ and $\lvert E_2 \rvert$.	*1*	*1*	**6**
		b)	Apply Kruskal's algorithm to find the minimal spanning tree of the graph given below 	*2*	*1*	**7**
		c)	Apply the Dijkstra's algorithm to find the shortest path and its weight from the vertex **1** to each of the vertices **2, 3, 4, 5, 6** in the weighted directed graph shown below. 	*2*	*1*	**7**
			OR			
	6	a)	If a tree T has two vertices of degree 2, four vertices of degree 3 and three vertices of degree 4, then find the number of leaves in T.	*1*	*1*	**6**
		b)	Define cut set of a graph and hence find the maximum flow between the vertices **A** and **R** by identifying the cut set of minimum capacity. 	*1*	*1*	**7**
		c)	Apply Dijkstra's algorithm to find the shortest path and its weight from the vertex **1** to each of the other vertices in the directed graph given below. 	*2*	*1*	**7**
			UNIT - IV			
	7	a)	Find the coefficients of (i). x^9y^3 in the expansion of $(2x - 3y)^{12}$, and (ii). xyz^2 in the expansion of $(2x - y - z)^4$.	*1*	*1*	**6**
		b)	Out of 1200 students, 582 took Economics, 627 took English, 543 took Mathematics, 217 took Economics and English, 307 took Economics and Mathematics, 250 took Mathematics and English and 222 took all three subjects. How many took none of the three?	*1*	*1*	**7**

		c)	Define Catalan numbers and write first 4 Catalan numbers. Apply the moves $R:(x,y)\rightarrow(x+1,y)$ and $U:(x,y)\rightarrow(x,y+1)$ to find the number of ways one can go from $(2,1)$ to $(7,6)$ and not rise above the line $y=x-1$?	*1*	*1*	**7**
			OR			
	8	a)	Find the coefficient of (i). x^0 in the expansion of $\left(3x^2-\frac{2}{x}\right)^{15}$, and (ii). $x^2y^2z^3$ in the expansion of $(3x-2y-4z)^7$.	*1*	*1*	**6**
		b)	In a survey of 200 musicians, it was found that 40 wore gloves on the left hand and 39 wore gloves on the right hand. If 160 wore no gloves at all, how many wore a glove on only the right hand? Only the left hand? On both hands?	*1*	*1*	**7**
		c)	Evaluate d_5, d_6, d_7 and d_8 where d_n is the number of derangements of n objects.	*1*	*1*	**7**
			UNIT - V			
	9	a)	Solve the recurrence relation $a_{n+1}=4a_n$, $a_0=3$ for $n\geq 0$.	*1*	*1*	**6**
		b)	Apply mathematical induction to prove that (n^3-n) is divisible by 3 $\forall n\in Z^+$.	*1*	*1*	**7**
		c)	Find the solution of the recurrence relation $a_{n+1}-a_n=3^n$, $a_0=1$ for $n\geq 0$ by the method of generating functions.	*1*	*1*	**7**
			OR			
	10	a)	Apply mathematical induction to show that $2^n>n^2$ $\forall n>4\in Z^+$.	*1*	*1*	**6**
		b)	Solve the recurrence relation $a_n+a_{n-1}-6a_{n-2}=0$, $a_0=-1$ and $a_1=8$ for $n\geq 2$.	*1*	*1*	**7**
		c)	Prove that every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps.	*1*	*1*	**7**

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