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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

June 2025 Semester End Main Examinations

Programme: B.E. **Semester: III**
Branch: AI and ML **Duration: 3 hrs.**
Course Code: 23MA3BSMML **Max Marks: 100**
Course: Mathematical Foundation for Machine Learning - 1

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

			CO	*PO*	Marks
		UNIT – 1			
1	a)	Find the rank and nullity of the linear map $L:\mathbb{R}^4 \to \mathbb{R}^3$ defined by $A=\begin{bmatrix}1&2&3&1\\1&3&5&2\\3&8&12&-3\end{bmatrix}$. Hence determine whether the mapping is i) one-one ii) onto. Justify for each case.	*1*	*1*	**6**
	b)	Let the matrix of $L:\mathbb{R}^3 \to \mathbb{R}^2$ with respect to the bases $S=\{v_1,v_2,v_3\}$ and $T=\{w_1,w_2\}$ be $A=\begin{bmatrix}1&2&1\\-1&1&1\end{bmatrix}$ where $v_1=(-1,1,0)$, $v_2=(0,1,1)$, $v_3=(1,0,-1)$, $w_1=(1,2)$ and $w_2=(1,-1)$. Compute coordinate vectors $[L(v_1)]_T$, $[L(v_2)]_T$, $[L(v_3)]_T$ and images $L(v_1)$, $L(v_2)$ and $L(v_3)$.	*1*	*1*	**7**
	c)	Design and implement the pseudocode to determine the number of floating-point operations (multiplications, additions) to compute the determinant of a 5×5 by the method of cofactors.	*2*	*1*	**7**
		OR			
2	a)	Apply elementary row transformation to compute the determinant of the matrix $A=\begin{bmatrix}2&2&0&1\\2&-1&3&0\\-1&0&2&-4\\1&-3&5&3\end{bmatrix}$.	*1*	*1*	**6**
	b)	Consider the matrix $A=\begin{bmatrix}1&3&1\\1&2&0\\0&1&1\end{bmatrix}$ which defines a linear operator on $\mathbb{R}^3$. Find the matrix of the linear transformation relative to the basis $S=\{u_1,u_2,u_3\}=\left\{\begin{bmatrix}1\\1\\1\end{bmatrix},\begin{bmatrix}0\\-1\\1\end{bmatrix},\begin{bmatrix}1\\2\\3\end{bmatrix}\right\}$.	*1*	*1*	**7**

	c)	Is the linear operator G on $\mathbb{R}^4$ defined by $G\begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} -x+y+3z+t \\ x+2y+3z+4t \\ 3y+6z+5t \\ -2x-y-3t \end{bmatrix}$ non-singular? Is it invertible? If not, find the basis of the subspace whose image is zero.	*1*	*1*	**7**
		UNIT – 2			
3	a)	Find G^2 and G^3 if the linear transformation $G:\mathbb{R}^3 \to \mathbb{R}^3$ is defined by $G(x,\ y,\ z) = (2x-y+3z, 2x+3y+z, 3x+y+z)$.	*1*	*1*	**6**
	b)	Determine whether the linear transformations F, G and H from R^3 to R^2 are linearly independent or not when they are defined by $F(a,\ b, c) = (2a+b,\ 3a+b+c)$, $G(a,\ b,\ c) = (3a+b-c,\ 2a-b)$ and $H(a,\ b,\ c) = (5a+2b-c,\ 5a+c)$.	*1*	*1*	**7**
	c)	Find the image of the triangle having vertices $A(1,2,1)$, $B(2,5,1)$ and $C(3,2,1)$ in the homogeneous coordinates under the sequence of transformations R, followed by S, followed by T where $R = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $S = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $T = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$. Plot the original and the transformed triangle.	*1*	*1*	**7**
		OR			
4	a)	If the linear maps $F:R^3 \to R^2$ and $G:R^2 \to R^3$ are defined by $F(x,y,z) = (x+y+z,\ x-y+z)$ and $G(x,y) = (x+y,\ x-y,\ 2x+3y)$, then find $F \circ G$ and $G \circ F$, if they exist.	*1*	*1*	**6**
	b)	Find a single matrix that defines a rotation of the plane through an angle $\frac{\pi}{4}$ about the origin, while at the same time moves the points to twice their distance from the origin. Hence find the image of the triangle $(0,0)$, $(1,0)$ and $(1,1)$. Plot the original image and the transformed image.	*1*	*1*	**7**
	c)	Determine the image of the triangle with vertices $(1,2),(2,8),(3,2)$ when transformed by the affine transformation $T(x,y) = (x+4, y+2)$. Plot the region and its image.	*1*	*1*	**7**
		UNIT – 3			
5	a)	If the inner product on the vector space $\mathbb{R}^3$ over a real field is given by $\langle u,v \rangle = v^T A u$ where $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 3 & 1 \\ -2 & 1 & 2 \end{bmatrix}$ then find the angle between $u = (1,3,-4)$ and $v = (4,-2,2)$.	*1*	*1*	**6**

	b)	Verify the parallelogram law $\|u+v\|^2+\|u-v\|^2=2\left(\|u\|^2+\|v\|^2\right)$ if $A=\begin{bmatrix}2 & 2\\ -1 & 3\end{bmatrix}$ and $B=\begin{bmatrix}2 & 1\\ 1 & -2\end{bmatrix}$ in an inner product space of matrices $M_{2\times 2}$ with the inner product $\langle A,B\rangle = Tr\left(B^T A\right)$.	1	1	**7**
	c)	Show that the ordered set $S=\{u_1,u_2,u_3,u_4\}$ where $u_1=(1,1,1,1)$, $u_2=(1,1,-2,0)$, $u_3=(-2,0,-1,3)$ and $u_4=(-5,7,1,-3)$ is an orthogonal basis of $\mathbb{R}^4$. Hence express $v=(1,2,-1,3)$ as a linear combination of the vectors of S.	1	1	**7**
		OR			
6	a)	Consider $f(t)=t+2$, $g(t)=3t-2$ and $h(t)=2t-3$ in $P(t)$ with the inner product defined as $\langle p,q\rangle=\int_0^1 p(t)q(t)\,dt$. Find $\langle f,h\rangle$, $\langle g,h\rangle$ and the angle between $f(t)$ and $g(t)$.	1	1	**6**
	b)	Write a Pseudo code to determine the condition number of the matrix with respect to the L_1-norm. Calculate the condition number of the coefficient matrix of the system $\begin{bmatrix}-1 & 2\\ 1 & 10^4\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}1\\ 10^4\end{bmatrix}$ with respect to L_1-norm and hence conclude that the problem is ill-conditioned.	1	1	**7**
	c)	Prove or disprove whether the mapping $\left(\mathbb{R}^2,\langle\ ,\ \rangle\right)$ is an inner product space if $\langle u,v\rangle=3u_1v_1-u_1v_2-u_2v_1+5u_2v_2$ where $u=(u_1,u_2)$ and $v=(v_1,v_2)$. Hence find the norm of $(-2,3)$ with respect to norm induced by the inner product.	1	1	**7**
		UNIT – 4			
7	a)	Let W be a subspace of $\mathbb{R}^5$ spanned by $u=(1,2,3,-1,2)$ and $v=(2,4,7,2,-1)$. Find a basis of $W^{\perp}$.	1	1	**6**
	b)	Find an orthogonal basis and hence an orthonormal basis of the subspace W spanned by $(1,1,1,1)$, $(1,1,2,4)$ and $(1,2,-4,-3)$ in $\mathbb{R}^4$.	1	1	**7**
	c)	A steel producer gathers the following data. Year: 1997, 1998, 1999, 2000, 2001 Annual Sales (million \$): 1.2, 2.3, 3.2, 3.6, 3.8 Represent the years 1997,…2001 as 0, 1, 2, 3, 4 respectively and let x denote the year and y denote the annual sales (in millions of dollars). Design and implement the pseudocode to determine the least squares line of the form $y=a+bx$. Hence predict the annual sales in the year 2007.	2	1	**7**
		OR			

Year	1997	1998	1999	2000	2001
Annual Sales (million $)	1.2	2.3	3.2	3.6	3.8

	8	a)	Find the projection of the vector $v=(1,2,3,4,6)$ along $W=span\{(1,2,1,2,1),(1,-1,2,-1,1)\}$ in $\mathbb{R}^5$.	*1*	*1*	**6**
		b)	Determine an orthogonal matrix whose first row is $\left(\frac{1}{3},\frac{-2}{3},\frac{2}{3}\right)$.	*1*	*1*	**7**
		c)	Let $C(K)$ denote the set of continuous functions $f:K\to\mathbb{R}$ where $K\subset\mathbb{R}$ is a closed and bounded interval. For two functions $f,\ g\in C(K)$, define $d(f,g)=\max_{x\in K}\lvert f(x)-g(x)\rvert$. Then show that function $d:\mathrm{C}(K)\times C(K)\to\mathbb{R}$ is a metric and the space $(C(K),d)$ is a metric space.	*1*	*1*	**7**
			UNIT – 5			
	9	a)	Find the minimal polynomial of the matrix $\begin{bmatrix}3 & 1 & -1\\ 2 & 4 & -2\\ -1 & -1 & 3\end{bmatrix}$.	*1*	*1*	**6**
		b)	Design and implement a pseudocode to compute spectral norm of AA^T if $A=\begin{bmatrix}2 & 2 & 1\\ 2 & -1 & 0\\ 1 & 0 & 1\end{bmatrix}$.	*2*	*1*	**7**
		c)	Determine the orthogonal matrix that diagonalizes the matrix A^TA when $A=\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & -1\end{bmatrix}$.	*1*	*1*	**7**
			OR			
	10	a)	Obtain the eigen space of the linear transformation $T:P_2(t)\to P_2(t)$ given by $T\ (at+b)=(2a-b)t+(-a+2b)$.	*1*	*1*	**6**
		b)	Compute A^5 when $A=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$ by using eigenvalue decomposition.	*1*	*1*	**7**
		c)	Find the characteristic and minimal polynomial of the matrix $\begin{bmatrix}4 & -1 & 0 & 0 & 0\\ 1 & 2 & 0 & 0 & 0\\ 0 & 0 & 3 & 1 & 0\\ 0 & 0 & 0 & 3 & 1\\ 0 & 0 & 0 & 0 & 3\end{bmatrix}$.	*1*	*1*	**7**

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