U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

January / February 2025 Semester End Main Examinations

Programme: B.E. **Semester: III**
Branch: Computer Science and Business Systems **Duration: 3 hrs.**
Course Code: 24MA3BSPBS **Max Marks: 100**
Course: Probability Theory for Business Systems

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.
3. Use of statistical tables are permitted.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	***CO***	***PO***	**Marks**
1	a)	Find the probability of getting a total of 7 or 11 when a pair of dice is tossed.	*1*	*1*	**6**
	b)	The probability that a regularly scheduled flight departs on time is 0.83, the probability that it arrives on time is 0.82 and the probability that it departs and arrives on time is 0.78. Find the probability that a plane (i) arrives on time, given that it did not depart on time (ii) depart on time, given that it did not arrive on time.	*2*	*1*	**7**
	c)	In a certain assembly plant, three machines, A, B and C make 30%, 45% and 25% of the products. It is known that 2%, 3% and 2% of the products made by each machine are defective respectively. (i) Find the probability that the product is defective (ii) If a product was chosen randomly and found to be defective, what is the probability that it was made by machine C?	*2*	*1*	**7**
		OR			
2	a)	If the probabilities are 0.09, 0.15, 0.21 and 0.23 that a person purchasing a new automobile will choose the color green or white or red or blue respectively, find the probability that a given buyer will purchase a new automobile that comes in one of those colors?	*1*	*1*	**6**
	b)	A town has two fire engines which operates independently. The probability that a specific engine is available when needed is 0.96. Find (a) the probability that neither is available when needed (b) the probability that fire engines are available when needed.	*2*	*1*	**7**
	c)	A federal agency employs three consulting firms A, B and C with probabilities 0.40, 0.35 and 0.25 respectively. From past experience it is known that the probability of cost over runs for the firms are 0.05, 0.03 and 0.15 respectively. (i) What is the probability that the consulting firm involved is company C? (ii) What is the probability that it is company A?	*2*	*1*	**7**

<table>
<tr><td></td><td></td><td></td><td>UNIT - 2</td><td></td><td></td><td></td></tr>
<tr><td></td><td>3</td><td>a)</td><td>Calculate the mean and variance of $g(X) = 2X + 3$, where X is a random variable with probability mass function:
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>$f(x)$</td><td>0.25</td><td>0.125</td><td>0.5</td><td>0.125</td></tr>
</table></td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>A random variable X has the following probability distributions:
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>$P(x)$</td><td>0</td><td>k</td><td>$2k$</td><td>$2k$</td><td>$3k$</td><td>k^2</td><td>$2k^2$</td><td>$7k^2+k$</td></tr>
</table>
Find (i) k (ii) $P(x>6)$ (iii) Mean and the probability distribution.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>In a certain factory turning out of razor blades, there is a small probability of 1/500 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) no defective (ii) one defective (iii) two defective blades in a consignment of 10,000 packets.</td><td>2</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>4</td><td>a)</td><td>Find the expected value of $Y = (X-1)^2$, where X is a random variable with probability mass function:
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>$f(x)$</td><td>1/3</td><td>1/2</td><td>0</td><td>1/6</td></tr>
</table></td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>If 3% of the product produced by a machine is found to be defective. Find the probability that first defective occurs in the
(i) 5^{th} item inspected (ii) first five items inspected</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>The number of industrial injuries per working week in a particular factory is known to follow a Poisson distribution with mean 0.5. Find the probability that (i) in a particular week there will be less than 2 accidents, (ii) in a particular week there will be more than 2 accidents; (iii) in three weeks period there will be no accidents.</td><td>2</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 3</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>The weekly demand for a certain drink, in thousands of litres, at a chain of convenience store is a continuous random variable $g(X) = X^2 + X - 2$, where X has the density function:
$f(x) = \begin{cases} 2(x-1), & 1 < x < 2 \\ 0, & Otherwise \end{cases}$.
Find the expected value of weekly demand of drink.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>In an examination 7% of students score less than 35% marks and 89% of students score less than 60% marks, Find the mean and standard deviation, if the marks are normally distributed.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>The sale per day in a shop is exponentially distributed with average sale amounting to Rs.100 and net profit is 8%. Find the probability that the net profit exceeds Rs.30 on a day.</td><td>2</td><td>1</td><td>7</td></tr>
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<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>6</td><td>a)</td><td>Let X be a continuous random variable with density function $f(x)=\begin{cases}\frac{x^2}{3}, -1<x<2,\\ 0, \quad elsewhere.\end{cases}$ Find the variance of the random variable $g(X)=4X+3$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>The mean weight of 1,000 students during medical examination was found to be 70kg and Standard Deviation weight 6. Assume that the weight is normally distributed, find the number of students having weight (i) less than 65kg (ii) more than 75kg (iii) between 65kg to 75kg.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Derive an expression for mean and variance of Erlang distribution.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 4</td><td></td><td></td><td></td></tr>
<tr><td></td><td>7</td><td>a)</td><td>Find the constant k such that $f(x)=\begin{cases}kx^2, 0<x<3,\\ 0, \quad otherwise.\end{cases}$ is a probability density function. Also compute the following:
$(i)\ P(1<x<2)$
(ii) $P(x\le 1)$</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>The joint probability distribution of two random variables X and Y are given as follows:
<table>
<tr><td>X \ Y</td><td>-2</td><td>-1</td><td>4</td><td>5</td></tr>
<tr><td>1</td><td>0.1</td><td>0.2</td><td>0</td><td>0.3</td></tr>
<tr><td>2</td><td>0.2</td><td>0.1</td><td>0.1</td><td>0</td></tr>
</table>
Find (i) E(X) (ii)E(Y) (iii) Cov (X,Y)</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>The joint density function of two continuous random variables X and Y is given by:
$f(x,y)=\begin{cases}\frac{xy}{96}, 0\le x\le 4, 1<y<5,\\ 0, \quad otherwise.\end{cases}$
Show that X and Y are independent.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>8</td><td>a)</td><td>If X and Y are continuous random variables having joint density function
$f(x,y)=\begin{cases}c(x^2+y^2), & 0\le x\le 1, 0\le y\le 1,\\ 0, & otherwise.\end{cases}$
Determine (i) the value of c (ii) $P(x<\frac{1}{2}, y>\frac{1}{2})$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Suppose that two batteries are randomly chosen without replacement from the following group of 12 batteries. 3 new batteries, 4 used (working) batteries, 5 defective batteries. If X is the random variable that denotes the number of new batteries chosen and Y is the random variable that denote the number of used batteries chosen. Find the joint probability distribution of X and Y. Also find $COV(X,Y)$.</td><td>2</td><td>1</td><td>7</td></tr>
</table>

		c)	The joint density function of two continuous random variables X and Y is given by $f(x,y) = \begin{cases} 2, & 0 < x < y < 1, \\ 0, & otherwise. \end{cases}$ Find the covariance between x and y.	*1*	*1*	**7**
			UNIT - 5			
	9	a)	A salesman's territory consists of 3 cities A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in city B. However, if he sells in either B or C, then the next day he is twice as likely to sell in city A as in other city. (i) Find the transition matrix P (ii) In the long run, how often does he sell in each of the cities.	*2*	*1*	**6**
		b)	Each year a man trades his car for a new car. If he has a 'MARUTI' he trades it for 'AMBASSADOR'. If he has a 'AMBASSADOR' he trades it for a 'SANTRO'. If he has a 'SANTRO' he is just as likely to trade it for new 'SANTRO' or for a new 'MARUTI' or a new 'AMBASSADOR' one. In 2000, he bought his first car which was 'AMBASSADOR'. Find the probability that he has in (i) 2002 MARUTI (ii) 2002 SANTRO (c) 2003 AMBASSADOR.	*2*	*1*	**7**
		c)	A housewife buys 3 kinds of cereals A, B and C. She never buys the same cereal in successive weeks. If she buys cereal A, the next week she buys cereal B. However, if she buys cereal B or C, the next week she is three times as likely to buy cereal A as the other cereal. In the long run, how often she buys each of the three cereals?	*2*	*1*	**7**
			OR			
	10	a)	Define irreducible state of Markov chain. Show that the Markov chain whose transition probability matrix P= $\begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ is irreducible.	*1*	*1*	**6**
		b)	Three boys A, B and C are throwing ball to each other. A always throws ball to B and B always throws the ball to C. C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball find the probabilities that after three throws (i) A has the ball (ii) B has the ball (iii) C has the ball.	*2*	*1*	**7**
		c)	A gambler's luck follows a pattern. If he wins a game, the probability of winning the next game is 0.6. However, if he loses a game, the probability of losing the next game is 0.7. There is an even chance of gambler winning the first game. If so, (i) What is the probability of he winning the second game? (ii) What is the probability of he winning the third game? (iii)In the long run, how often he will win?	*2*	*1*	**7**

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