U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

June 2025 Semester End Main Examinations

Programme: B.E. **Semester: III**
Branch: Computer Science and Business Systems **Duration: 3 hrs.**
Course Code: 24MA3BSPBS **Max Marks: 100**
Course: Probability Theory for Business Systems

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.
3. Use of statistical tables are permitted.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	*CO*	*PO*	Marks
1	a)	Three groups of children contain 3 girls and 1 boy, 2 girls and 2 boys 1 girl and 3 boys respectively. One child is selected at random from each group. Find the probability of selecting 1 girl and 2 boys.	*1*	*1*	**6**
	b)	In a bolt factory there are four machines A, B, C and D manufacturing 20%,15%,25% and 40% of the total production respectively. Out of these 5%,4%, 3% and 2% are defective. If a bolt drawn at random was found defective, what is the probability that it was manufactured by A or D.	*1*	*1*	**7**
	c)	An automobile dealer offers vehicles with the following options: (i) With or without automatic transmission, (ii) With or without air-conditioning, (iii) With one of two choices of a stereo system, (iv) With one of three exterior colors. If the sample space consists of the set of all possible vehicle types, what is the number of outcomes in the sample space? Explain through the tree diagram.	*2*	*1*	**7**
		OR			
2	a)	A bag contains 10 white and 3 red balls while another bag contains 3 white and 5 red balls. 2 balls are drawn at random from the first bag and put in the second bag. Then a ball is drawn at random from the second bag. What is the probability that it is a white ball?	*1*	*1*	**6**
	b)	In a school 25% of the students failed in first language, 15% of the students failed in second language and 10% of the students failed in both. If a student is selected at random find the probability that (i) He failed in first language if he had failed in the second language. (ii) He failed in second language if he had failed in the first language. (iii)He failed in either of the two languages.	*1*	*1*	**7**
	c)	A shooter can hit a target in 3 out of 4 shots and another shooter can hit the target in 2 out of 3 shots. Analyze the probability that the target is being hit (i)When both of them try (ii) By only one shooter	*2*	*1*	**7**

			UNIT - 2			
	3	a)	A random variable X has the following function for various values of x $\begin{array}{|c|c|c|c|c|c|c|c|c|} \hline x & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline P(x) & 0 & k & 2k & 2k & 3k & k^2 & 2k^2 & 7k^2+k \\ \hline \end{array}$ (i) Find k (ii) Evaluate $P(x<6)$ and $P(x \geq 6)$.	*1*	*1*	**6**
		b)	Derive the mean and variance of Poisson's distribution.	*1*	*1*	**7**
		c)	What is the probability that the marketing representative may select (i) more than 6 people (ii) exactly 6 people, before he finds one who attended the lost home. Given cumulative distribution function of a geometric random variable with $(1-p)=0.8$.	*1*	*1*	**7**
			OR			
	4	a)	X is a discrete random variable having $P(x)$ defined as follows: $P(x)=\begin{cases} \frac{x}{15}, & 1 \leq x \leq 5, \\ 0, & x>5. \end{cases}$. Show that $P(x)$ is a probability function and find $P\left(\frac{1/2<X<5/2}{X>1}\right)$.	*1*	*1*	**6**
		b)	Derive an expression for the mean and variance of geometric distribution.	*1*	*1*	**7**
		c)	2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses contains (i) No defective fuses (ii) 3 or more defective fuses.	*2*	*1*	**7**
			UNIT - 3			
	5	a)	Is the function $f(x)=\begin{cases} e^{-x}, x \geq 0 \\ 0, \quad x<0 \end{cases}$ a probability density function? If so, determine the probability that the variate having this density will fall in the interval (1, 2).	*1*	*1*	**6**
		b)	Derive an expression for the mean and variance of an exponential distribution.	*1*	*1*	**7**
		c)	The marks of 1000 students in an examination follow a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be (i) Less than 65 (ii) More than 75 and (iii) Between 65 and 75.	*1*	*1*	**7**
			OR			
	6	a)	Find the constant k such that $f(x)=\begin{cases} kx^2, 0<x<3, \\ 0, \quad otherwise. \end{cases}$ is a probability density function. Also compute the following: (i) $P(1<x<2)$ (ii) $P(x \leq 1)$	*1*	*1*	**6**
		b)	Derive an expression for the mean and variance of Normal distribution.	*1*	*1*	**7**
		c)	The length of a telephone conversation in a booth follows an exponential distribution with an average to be 5 minutes. In this booth, find the probability that a random call must: (i) ends less than 5 minutes (ii) between 5 and 10 minutes	*2*	*1*	**7**

<table>
<tr><td></td><td></td><td></td><td>UNIT - 4</td><td></td><td></td><td></td></tr>
<tr><td></td><td>7</td><td>a)</td><td>The joint probability distribution table for two random variables X and Y is as follows.
<table>
<tr><td>Y / X</td><td>-2</td><td>-1</td><td>4</td><td>5</td></tr>
<tr><td>1</td><td>0.1</td><td>0.2</td><td>0</td><td>0.3</td></tr>
<tr><td>2</td><td>0.2</td><td>0.1</td><td>0.1</td><td>0</td></tr>
</table>
Determine the marginal probability of X and Y. Also, find Expectations of X, Y and XY.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>X and Y are random variables having joint density function $f(x,y) = \begin{cases} 4xy, 0 \le x \le 1, 0 \le y \le 1 \\ 0, \quad otherwise \end{cases}$. Verify that
(i) $E(X+Y) = E(X) + E(Y)$ (ii) $E(XY) = E(X).E(Y)$</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>The joint density function of two continuous random variables X and Y is given by:
$f(x,y) = \begin{cases} \frac{xy}{96}, 0 \le x \le 4, 1 < y < 5, \\ 0, \quad otherwise. \end{cases}$
Show that X and Y are independent and find the value of $E(2X+3Y)$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>8</td><td>a)</td><td>If X and Y are independent random variables, X take values 2,5,7 with probability 1/2,1/4,1/4 respectively. Y take values 3,4,5 with the probability 1/3, 1/3, 1/3 respectively.
(i) Find the joint probability distribution of X and Y.
(ii) Find the probability distribution of $Z = X + Y$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Given the following joint distribution of the random variables X and Y, find the corresponding marginal distribution. Also compute the covariance and the correlation of the random variables X and Y.
<table>
<tr><td>Y / X</td><td>1</td><td>3</td><td>9</td></tr>
<tr><td>2</td><td>1/8</td><td>1/24</td><td>1/12</td></tr>
<tr><td>4</td><td>1/4</td><td>1/4</td><td>0</td></tr>
<tr><td>6</td><td>1/8</td><td>1/24</td><td>1/12</td></tr>
</table></td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>If the joint probability function of the random variables X and Y is given by $f(x,y) = \begin{cases} cxy, \quad 0 \le x \le 2, \ 0 \le y \le x \\ 0, \quad Otherwise \end{cases}$. Determine c and hence find $P(1/2 < X < 1)$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 5</td><td></td><td></td><td></td></tr>
<tr><td></td><td>9</td><td>a)</td><td>If $A = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}$ is a stochastic matrix and $v = [v_1, v_2]$ is a probability vector, show that vA is also a probability vector.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find the unique fixed probability vector of the regular stochastic matrix P as given
$P = \begin{bmatrix} 0 & 1/2 & 1/4 & 1/4 \\ 1/2 & 0 & 1/4 & 1/4 \\ 1/2 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \end{bmatrix}.$</td><td>1</td><td>1</td><td>7</td></tr>
</table>

		c)	A man's smoking habits are as follows. If he smokes filter cigarettes one week, he switches to non-filter cigarettes the next week with probability 0.2. On the other hand, if he smokes non-filter cigarettes one week there is a probability of 0.7 that he will smoke non filter cigarettes the next week as well. In the long run, how often does he smoke filter cigarettes?	*2*	*1*	**7**
			OR			
	10	a)	Find the unique fixed probability vector of the regular stochastic matrix $A = \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix}$.	*1*	*1*	**6**
		b)	The transition probability matrix of a Markov chain is given by $P = \begin{bmatrix} 1/2 & 0 & 1/2 \\ 1 & 0 & 0 \\ 1/4 & 1/2 & 1/4 \end{bmatrix}$ and initial probability distribution is $P^0 = \left(\frac{1}{2}, \frac{1}{2}, 0\right)$. *Find* P_{13}^2, P_{23}^2, P^2 and P_1^2.	*1*	*1*	**7**
		c)	A gambler's luck follows a pattern. If he wins a game, the probability of winning the next game is 0.6. However, if he loses a game, the probability of losing the next game is 0.7. There is an even chance of gambler winning the first game. If so, (i) What is the probability of he winning the second game? (ii) What is the probability of he winning the third game?	*2*	*1*	**7**

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