B.M.S. COLLEGE OF ENGINEERING, BENGALURU – 560 019
Autonomous college, affiliated to VTU
DEPARTMENT OF MATHEMATICS

STATISTICS AND DISCRETE MATHEMATICS
(Course Code: 23MA3BSSDM)

UNIT-2: PROBABILITY DISTRIBUTIONS

Poisson Distribution

$X \sim P(\lambda)$

$p(x) = \frac{e^{-\lambda x} \lambda^x}{x!}; x = 0, 1, 2, 3, ...$

1. The traffic police recorded an average of 3 road accidents per week. The number of accidents is distributed according to a Poisson distribution. Calculate the probability in any week of exactly 2 accidents.

2. Alpha particles are emitted by radioactive source at the rate of three per every minute on the average. The number of particles is distributed according to the Poisson distribution. Calculate the probability of getting exactly 5 emissions in one minute.

3. A distributor of bean seeds determines from extensive tests that 5% of large batch of seeds will not germinate. He sells the seeds in packets of 200 and guarantees 98% germination. Determine the probability that a particular packet will violate the guarantee.

4. It is known from past experience that in a certain plant there are on the average 4 industrial accidents per year. Find the probability that in a given year there will be less than 4 accidents.

5. In a town, 10 accidents take place in a span of 50 days. Assuming that the number of accidents follows the Poisson distribution, find the probability that there will be 3 or more accidents in a day.

6. A book contains 100 misprints distributed randomly throughout its 100 pages. Assuming Poisson distribution, find the probability that a page observed at random contains at least two misprints.

7. A switch board can handle only 4 telephone calls per minute. If the incoming calls per minute follow a Poisson distribution with parameter 3, find the probability that the switchboard is over taxed in any one minute.

8. A car hire firm has two cars which it hires out day by day. The number of demands for acar on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days on which (i) there is no demand (ii) demand is refused.
9. A shop has 4 diesel generator sets which it hires every day. The demand for a generator set onan average is a Poisson variate with value 5/2. Obtain the probability that on a particular day (i) There was no demand (ii) A demand is refused.

10. A communication channel receives independent pulses at the rate of 12 pulses per micro second. The probability of transmission error is 0.001 for each micro second. Compute the probabilities of (i) no error during a micro second (ii) one error per micro second (iii)atleast one error per micro second (iv) two errors (v) At most two errors.

11. The probability that a news reader commits no mistake in reading the news is e^{-3} . Find the probability that on particular news broadcast he commits (i) only 2 mistakes (ii) more than 3 mistakes (iii) at most 3 mistakes.

12. In a certain factory turning out razors blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distributionto calculate the approximate number of packets containing (i) no defective (ii) one defective and (iii) two defective blades respectively in a consignment of 10,000 packets.

13. The number of accidents in a year to taxi in a city follows a Poisson distribution withmean 3. Out of 1000 taxi drivers find approximately the number of the drivers with (i) no accident in a year (ii) more than 3 accidents in a year.

14. 2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses contains (i) No defective fuses (ii) 3 or more defective fuses.

15. If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals, (i) exactly 3 (ii) more than 2 will suffer a bad reaction.

16. If X follows a Poisson law such that $P(X=2)=\frac{2}{3}\ P(X=1)$, find P(X=0) and P(X=3).

17. If x is a Poisson variate such that $P(x=2)=9\ P(x=4\ )+90P(x=6\ )$, compute mean and variance of the Poisson distribution.

18. Fit a Poisson distribution for the following data and calculate the theoretical frequencies

x	0	1	2	3	4
f	111	63	22	3	1

19. Fit a Poisson distribution for the following frequency distribution.

x	0	1	2	3	4
f	46	38	22	9	1

20. Fit a Poisson distribution for the following frequency distribution.

x	0	1	2	3	4
f	122	60	15	2	1

21. The frequency of accidents per shift in a factory is as shown in the following table:

Accidents per shifts	0	1	2	3	4
Frequency	180	92	24	3	1

Calculate the mean number of accidents per shifts and the corresponding Poisson distribution and compare with actual observations.

22. The number of accidents per day (x) as recorded in a textile industry over a period of 400days is given. Fit a Poisson distribution for the data and calculate the theoretical frequencies.

x	0	1	2	3	4	5
f	173	168	37	18	3	1

Geometric Distribution

$X \sim G(p)$

$p(x) = pq^{x-1}; x = 1, 2, 3, \ldots\ldots$

1. The probability that a phone call leads to a sale is 0.4. Calculate the probability that the first sale occurs in the fifth call.

2. In a certain town 30% of the people own an iPhone. A researcher asks people at random whether they own an iPhone. The random variable X represents the number of people asked up to and including the first person who owns an iPhone. Determine the chances that:
   (i) 4 person owns an iPhone
   (ii) at least 3 people own an iPhone
   (iii) at most 4 people own an iPhone

3. A representative from the National Football League's Marketing Division randomly selects people on a random street in Kansas City, Kansas until he finds a person who attended the last home football game. Let the probability that he succeeds in finding such a person is 0.20. And, let X denote the number of people he selects until he finds his first success.
   (i) What is the probability that the marketing representative must select 4 people before he finds one who attended the last home football game?
   (ii) What is the probability that the marketing representative must select more than 6 people before he finds one who attended the last home football game?
   (iii) How many people should we expect (that is, what is the average number) the marketing representative needs to select before he finds one who attended the last home football game? And, while we're at it, what is the variance?

4. An old lawn mower has a 20% chance of starting on a particular pull. Find the probability that it takes (i) exactly 3 pulls to start the mower (ii) 10 or fewer pulls to start the mower

5. A person decides to continue placing a bet of Rs.5000 on the number 5 in consecutive spins of a roulette wheel until he wins. On any spin there is a 1 on 50 chances that the ball would land on the number 5.
   (i) How many spins do you expect until he wins?
   (ii) What is the amount he is expected to spend until he has his first win?
   (iii) What are the chances that it takes 5 spins before he wins?
   (iv) What are the chances that it would take him more than 50 chances to win?

6. To finish a board game, a person A needs a sum of 4 with two dice. What is the probability that it takes A under 5 tries to win? How many rolls would you expect A to take until she wins?

7. A practicing shooter scores 93% of his shots during a training session. What are the chances that he would not miss a single shoot till his 20^{th} try? What is the expected number of shots taken before his first miss?

Uniform Distribution (Continuous)

$$X \sim U(a,b)$$

$$f(x)=\begin{cases} \dfrac{1}{b-a} & a \le x \le b \\ o & otherwise \end{cases}$$

1. If X is uniformly distributed in $-2 \le X \le 2$, find (i) $P(X < 1)$ (ii) $P\left(|X-1| \ge \frac{1}{2}\right)$.

2. If X is uniformly distributed with mean 1 and variance$\frac{4}{3}$, find $P(X < 0)$.

3. A random variable X has a uniform distribution $U[-3,3]$. Find k if $P(X > 3) = k$.

4. A point is chosen at random from the line segment$[0,2]$. What is the probability that the chosen point lies (a) $1 \le X \le \frac{3}{2}$ (b) $X \le 1$ (c) $X \ge 3$

5. If X is uniformly distributed in $[-\alpha, \alpha]$ with $\alpha > 0$ such that $P(X > 1) = \frac{1}{3}$ then determine α.

6. A bus travels between two cities A and B which are 100 miles apart. If the bus has a breakdown, the distance X of the point of breakdown from the city A has a uniform distribution$U[0,100]$. There are service garages in the city A, city B and midway between the two cities such that in case of a breakdown a tow truck is sent from the garage nearest to the point of breakdown.
   (i) What is the probability that the tow truck has to travel more than 10 miles to reach the bus?
   (ii) Would it be more “efficient” if the three service garages were placed at 25, 50 and 75 miles from city A, apart from service garages at city A and city B?

7. If a conference room cannot be reserved for more than 4 hours, find the probability that a given conference lasts more than 3 hours.

8. The daily amount of coffee (in liters) dispensed by a machine is uniformly distributed with $a = 7$ and $b = 10$. Determine the probability that the amount of coffee dispensed by the machine will be (i) at most 8.8 liters (ii) more than 7.4 liters but less than 9.5 liters (iii) a least 8.5 liters.

9. The driving time X from a person’s home to the train station is uniformly distributed as $U[10,50]$. If it takes 2 minutes to board the train, determine the probability that the person caches the 7 am train if he starts at 6:43am from his home.

10. A bus arrives every 10 minutes at a bus stop. Assuming that the waiting time X for a bus is uniformly distributed, find the probability that the person has to wait for the bus (i) for more than 7 minutes (ii) between 2 and 7 minutes

11. The amount charged for a visit to a dental clinic is uniformly distributed from 0 to 1000 (in INR). Given that the amount charged for a visit exceeds Rs 500, calculate the probability that it exceeds Rs. 750.

Exponential Distribution

$X \sim E(\alpha)$

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & x > 0, \alpha > 0 \\ 0 & otherwise \end{cases}$$

1. In a certain town, the duration of shower is exponentially distributed with mean 5 minutes. What is the probability that the shower will last for (i) less than 10 minutes (ii) 10 minutes or more and (iii) between 10 and 12 minutes.

2. The length of a telephone conversation in a booth is exponentially distributed and found on an average to be 5 minutes. Find the probability that the random call made from this booth (i) Ends in less than 5 minutes (ii) Between 5 and 10 minutes.

3. At a certain city bus stop, three buses arrive per hour on an average. Assuming that the time between successive arrivals is exponentially distributed; find the probability that the time between the arrivals of successive buses is (i) less than 10 minutes (ii) at least 30 minutes.

4. The sales per day in a shop is exponentially distributed with average sale amounting to Rs100/- and net profit is 8%. Find the probability that the net profit exceeds Rs. 30/- on2 consecutive days.

5. The daily turnover in a medical shop is exponentially distributed with Rs.6000 as the average with a net profit of 8%. Find the probability that the net profit exceeds Rs.500on a randomly chosen day.

6. Increase in sales per day in a shop is exponentially distributed with mean of Rs.600/-. Sales tax is to be levied at 9%. What is the probability that sales tax will exceed Rs.81per day?

7. The life of a compressor manufactured by a company is known to be 200 months on an average following an exponential distribution. Find the probability that the life of a compressor of that company is (i) Less than 200 months (ii) Between 100 months and 25years?

8. The time X (seconds) that it takes a certain online computer terminal (the elapsed time between the end of user's inquiry and the beginning of the system's response to that inquiry) has an exponential distribution with expected time 20 seconds. Compute the probabilities (a) $P(X \le 30)$ (b) $P(X \ge 20)$ (c) $P(20 \le X \le 30)$.

9. Let the mileage (in thousands of miles) of a particular tyre be a random variable X having the probability density $f(x) = \begin{cases} \frac{1}{20} e^{-x/20} & x > 0 \\ 0 & x \le 0 \end{cases}$.Find the probability that one of these tyres will last (i) at most 10,000 miles (ii) anywhere from16,000 to 24,000 miles (iii) at least 30,000 miles (iv) mean (v) variance

Normal Distribution/ Gaussian Distribution

$$X \sim N\left(\mu, \sigma^2\right)$$

$$f\left(x\right) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)^2} ; -\infty < x, \mu < \infty$$

1. In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and S.D of 60 hours. estimate the number of bulbs likely to burn for (a) More than 2150 hours (b) Less than 1950 hours(c) More than 1920 hours and but less than 2160 hours.

2. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find themean and S.D. of the distribution.

3. If the total cholesterol values for a certain population are approximately normally distributed with a mean of 200mg/ml and standard deviation of 20mg/ml. Find the probability that an individual selected at random from this population will have a cholesterol value:
   i. Between 180 and 200mg/ml.
   ii. Greater than 225mg/ml.
   iii. Less than 150mg/ml.

4. In a certain examination, the percentage of candidates passing and getting distinctions were 45 and 9 respectively. Estimate the average marks obtained by the candidates, the minimum pass and distinction marks being 40 and 75 respectively.

5. A manufacturer of air-mail envelopes knows from experience that weight of the envelopes is normally distributed with mean 1.95 gm and S.D. 0.05 gm. About how many envelopes weighting (i) 2 gm or more (ii) 2.05 gm or more can be expected in a given packet of 100 envelopes.

6. The mean height of 500 students is 151 cm and the S.D. is 15 cm. Assuming that the heights are normally distributed, find how many student's heights lie between 120 and 155 cm.

7. The mean and S.D. of the marks obtained by 1000 students in an examination are respectively 34.4 and 16.5. Assuming the normality of the distribution, find the approximate number of students expected to obtain marks between 30 and 60.

8. In a examination taken by 500 candidates, the average and S.D. of marks obtained (normally distributed) are 40% and 10%. Find approximately (i) how many will pass, if 50% is fixed as a minimum? (ii)What should be the minimum if 350 candidates are to pass? (iii) How many have scored marks above 60%?

9. The mean inside diameter of a sample of 200 washers produced by a machine is 5.02 mm and the S.D. is 0.05 mm. the purpose for which these washers are intended allows a maximum tolerance in the diameter of 4.96 to 5.08 mm, otherwise the washers are considered defective. Determine the percentage of defective washers produced by the machine, assuming the diameters are normally distributed.

10. It is given that the age of thermostats of particular makes follow the normal law with mean 5 years and S.D. 2 years. 1000 units are sold out every month. How many of them will have to be replaced at the end of the second year?

11. The income of a group of 10,000 persons was found to be normally distributed with mean Rs. 750 p.m. and S.D. of Rs. 50. Show that, of this group, about 95% had income exceeding Rs. 668 and only 5% had income exceeding Rs. 832. Also find the lowest income among the richest 100.

12. If the top 15% of the students receives A grade and bottom 10% receives F grades in a mathematics examination, determine the (a) minimum marks to get an A grade (b) minimum mark to pass. Assume that the marks are normally distributed with mean 76 and standard deviation 15.

Gamma Distribution

$X \sim \Gamma(\alpha, \beta)$

$$f(x) = \begin{cases} \dfrac{x^{\alpha-1} e^{-x/\beta}}{\beta^{\alpha}\Gamma(\alpha)} & x > 0 \\ 0 & otherwise \end{cases}$$

1. The daily consumption of electric power (in million Kw-hours) in a certain city is a random variable X having the probability density

$$f(x) = \begin{cases} \frac{1}{9} x e^{-x/3}, if\ x > 0 \\ 0, if\ x \leq 0 \end{cases}$$

Find the probability that the power supply is adequate on any given day if the capacity of the power plant is 12 million Kw hours.

2. The lifetime X (in months) of a computer has a gamma distribution with mean 24 months and standard deviation 12 months. Find the probability that the computer will (i) last between 12 and 24 months (ii) last at most 24 months.

3. Suppose that the time (in hours) taken by a homeowner to mow his lawns is a random variable having a gamma distribution with parameters $\alpha = 2$ and $\beta = 2$. Find the probability that it takes (i) at most 1 hour (ii) at least 2 hours (iii) between 0.5 to 1.5 hours to mow the lawn.

4. The survival time (in weeks) of a male mouse exposed to radiation has a gamma distribution with $\alpha = 8$ and $\beta = 15$. Find the probability that the mouse survives (i) between 60 and 120 weeks (ii) at least 30 weeks. Find the mean and variance of X.

5. If a random variable has gamma distribution with $\alpha = 2$ and $\beta = 2$. Find (i) mean (ii) standard deviation (iii) the probability that X will take a value less than 4.

6. An actuary models the occurrence of claims from a portfolio of insurance policies. The time until the occurrence of the second claim is modelled by a gamma distribution with mean 10 minutes and variance 50 minutes. Determine the parameters of the distribution. Hence determine the probability that the time until the occurrence of the second claim exceeds 20 minutes.

7. Daily consumption of milk in a town in excess of 20,000 litres is approximately given by Gamma distribution with $\alpha = 3$ and $\beta = 10,000$. The town has a daily stock of 30,000 litres. Find the probability that the stock is insufficient on a given day.