U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

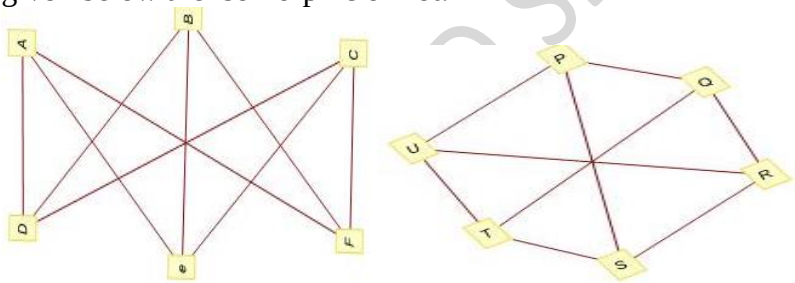
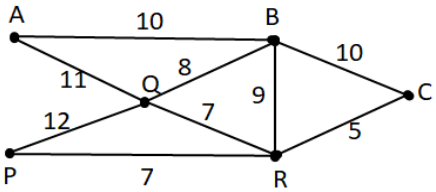
Autonomous Institute Affiliated to VTU

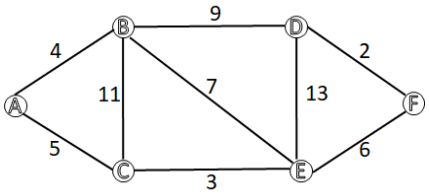
January / February 2025 Semester End Main Examinations

Programme: B.E. **Semester: III**
Branch: CSE/ISE **Duration: 3 hrs.**
Course Code: 19MA3BSSDM **Max Marks: 100**
Course: Statistics and Discrete Mathematics

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.
3. Use of Statistical tables is permitted.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

			UNIT - 1	*CO*	*PO*	Marks
1	a)		Determine the order $\|V\|$ of the graph G = (*V*, *E*) in the following cases (i) G has nine edges and all vertices have degree 3. (ii) G is regular with 15 edges.	*1*	*1*	**6**
	b)		Define isomorphism of two graphs. Determine whether the two graphs given below are isomorphic or not. 	*1*	*1*	**7**
	c)		Apply Kruskal's algorithm to find a minimal spanning for the weighted graph shown below. 	*1*	*1*	**7**
			OR			
2	a)		Obtain the incidence matrix for the graph whose adjacency matrix is given below. $\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array}\begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$	*1*	*1*	**6**

		b)	Prove that a connected graph G remains connected after removing an edge *e* from G if and only if *e* is a part of some cycle in G.	*1*	*1*	**7**
		c)	Apply Dijkstra's algorithm to obtain the shortest path from vertex A to each of the other vertices in the weighted network shown below.  	*1*	*1*	**7**
			UNIT - 2			
	3	a)	Find the coefficient of (i) x^9y^3 in the expansion of $(2x-3y)^{12}$. (ii) $a^2b^3c^2d^5$ in the expansion of $(a+2b-3c+2d)^{12}$.	*2*	*1*	**6**
		b)	In how many ways can the 26 letters of the English alphabets be permuted so that none of the pattern CAR, DOG, PUN or BYTE occurs?	*2*	*1*	**7**
		c)	Thirty students take a quiz. Then for the purpose of grading, the teacher asks the students to exchange papers. Find the probability that (i) No one is grading his own paper, (ii) Every student gets his own paper and (iii) Exactly one student gets his own paper.	*2*	*1*	**7**
			OR			
	4	a)	Find the coefficients of $x_1^2x_3x_4^3x_5^4$ in the expansion of $(x_1+x_2-x_3+2x_4-3x_5)^{10}$.	*2*	*1*	**6**
		b)	Determine the number of positive integers *n*, $1\le n\le 2000$ which are not divisible by two of the numbers $2,3,5 or 7$.	*2*	*1*	**7**
		c)	Define Catalan number. Hence, find in how many ways can one move from the point (3,8) to the point (11,16) in the $x-y$ plane by using only the *R* and *U* moves and without crossing the line $y=x+5$.	*2*	*1*	**7**
			UNIT - 3			
	5	a)	A communication channel receives independent pulses at the rate of 12 pulses per micro second. The probability of transmission error is 0.01 for each micro second. Compute the probabilities of (i) no error during a micro second (ii) atleast one error per micro second (iii) Atmost two errors.	*3*	*1*	**6**
		b)	In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D. of the distribution.	*3*	*1*	**7**
		c)	Two cards are selected at a random from a box which contains five cards numbered 1, 1, 2, 2 and 3. Find the joint distribution of X and Y where X denotes the sum and Y the maximum of the two numbers drawn. Also, determine $Cov(X,Y)$.	*3*	*1*	**7**
			OR			

	6	a)	Let the mileage (in thousands of miles) of a particular tyre be a random variable X having the probability density $f(x)=\begin{cases}\frac{1}{20}e^{-x/20} & x>0\\ 0 & x\le 0\end{cases}$. Find the probability that one of these tyres will last (i) at most 10,000 (ii) anywhere from 16,000 to 24,000 and (iii) at least 30,000 miles.	*3*	*1*	**6**
		b)	The number of computer malfunctions per day is recorded for 260 days with the following results. Number of malfunctions: 0, 1, 2, 3, 4, 5 Number of days: 77, 87, 55, 30, 5, 6 Fit a Poisson distribution.	*3*	*1*	**7**
		c)	The joint probability function of two random variable X and Y is given by $f(x,y)=k(2x+3y)$ for $0\le x\le 2; 1\le y\le 3$. (i) Find the constant k. (ii) Calculate marginal distribution of X and Y and (iii) Whether or not X and Y are independent.	*3*	*1*	**7**
			UNIT - 4			
	7	a)	A company claims that the mean thermal efficiency of diesel engines produced by them is 32.3. to test this claim, a random sample of 40 engines were examined which showed the mean thermal efficiency of 31.4 and standard deviation of 1.6. can the claim be accepted or not at 0.01 level of significance?	*3*	*1*	**6**
		b)	Random samples of specimens of coal from two mines A and B are drawn and their heat producing capacity (in millions of calories/ton) were measured yielding the following results: *Mine A*: 8350, 8070, 8340, 8130, 8260, – *Mine B*: 7900, 8140, 7920, 7840, 7890, 7950 Is there significant difference between the means of these two samples at 1% level of significance?	*3*	*1*	**7**
		c)	According to a theory in genetics, the proportion of beans of four types A, B, C and D in a generation should be 9:3:3:1. In an experiment, among 1600 beans, the frequency of beans of each of the above four types were 882, 313, 287, and 118 respectively. Does the result support the theory?	*3*	*1*	**7**
			OR			
	8	a)	Two samples of sizes 9 and 8 give the sum of squares of deviations from their respective means is equal to 160 inches^2 and 91 inches^2 respectively. Can these be regarded as drawn from the normal population with equal variances?	*3*	*1*	**6**
		b)	A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure: 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0 and 4. Can it be concluded that the stimulus will increase the blood pressure?	*3*	*1*	**7**
		c)	In a random sample of 100 tube lights produced by company A, the mean lifetime (mlt) of tube light is 1190 hours with standard deviation of 90 hours. Also, in a random sample of 75 tube lights from company B the mean lifetime is 1230 hours with standard deviation of 120 hours. Is there a difference between the mean lifetimes of the two brands of tube lights at a significance level of 0.05?	*3*	*1*	**7**

			UNIT - 5			
	9	a)	An auto insurance company classifies its customers in three categories: poor, satisfactory and preferred. No one moves from poor to preferred or from preferred to poor in one year. 40% of the customers in the poor category become satisfactory, 30% of those in the satisfactory category moves to preferred, while 10% become poor; 20% of those in the preferred category are downgraded to satisfactory. (a) Write the transition matrix for the model. (b) Check whether the transition matrix is a regular stochastic matrix.	*3*	*1*	**6**
		b)	In a certain city, the weather on a day is reported as sunny, cloudy or rainy. If a day is sunny, the probability that the next day is sunny is 70%, cloudy is 20% and rainy is 10%. If a day is cloudy, the probability that the next day is sunny is 30%, cloudy is 20% and rainy is 50%. If a day is rainy, the probability that the next day is sunny is 30%, cloudy is 30% and rainy is 40%. (i) If the Sunday is sunny, find the probability that the Wednesday is rainy (ii) Find the Steady State distribution.	*3*	*1*	**7**
		c)	The capacity of a communication line is 2000 bits per second. The line is used to transmit 8-bit characters. It is required to transmit a total of 12,000 characters per minute. Find the expected number of characters waiting to be transmitted and the average response time.	*3*	*1*	**7**
			OR			
	10	a)	Define irreducible Markov chain. Check whether the following Markov chain is irreducible. States | Rainy Tomorrow | Cloudy Tomorrow | Sunny Tomorrow Rainy Today | 1/2 | 1/3 | 1/6 Cloudy Today | 1/3 | 1/3 | 1/3 Sunny Today | 0 | 1/9 | 8/9	*3*	*1*	**6**
		b)	A computer device can be either in a busy mode (state 1) processing a task, or in an idle mode (state 2), when there are no tasks to process. Being in a busy mode, it can finish a task and enter an idle mode any minute with the probability 0.2. Thus, with the probability 0.8 it stays another minute in a busy mode. Being in an idle mode, it receives a new task any minute with the probability 0.1 and enters a busy mode. Thus, it stays another minute in an idle mode with the probability 0.9. The initial state is idle. Let X_n be the state of the device after n minutes. a) Find the distribution of X_2. b) Find the steady-state distribution of X_n.	*3*	*1*	**7**
		c)	At Bharat petrol pump, customers arrive according to a Poisson process with an average time of 5 minutes between arrivals. The service time is exponentially distributed with mean time equal to 2 minutes. On the basis of this information, find out: i) What would be the average number of customers in the queuing system? ii) What is the average time spent by a car in the petrol pump? iii) What is the average waiting time of a car before receiving petrol?	*3*	*1*	**7**

****