U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

September / October 2023 Supplementary Examinations

Programme: B.E. **Semester: III**
Branch: CSE/ISE **Duration: 3 hrs.**
Course Code: 19MA3BSSDM **Max Marks: 100**
Course: STATISTICS AND DISCRETE MATHEMATICS **Date: 12.09.2023**

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.
3. Use of Statistical tables is permitted.

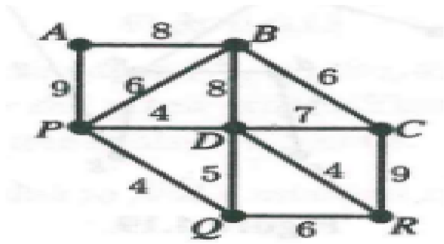
UNIT - I

1 a) Define a simple graph. Is there a simple graph with 1, 1, 3, 3, 3, 4, 6, 7 as the degrees of its vertices? Justify. **6**

b) Draw the graph and write its adjacency matrix whose incidence matrix is as given below. **7**

$$A(G) = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

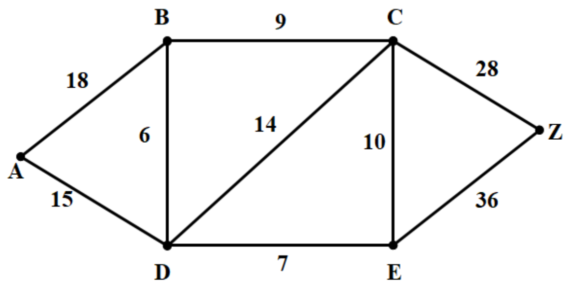
c) Apply Kruskal's algorithm to find the minimal spanning tree for the weighted graph shown below: **7**



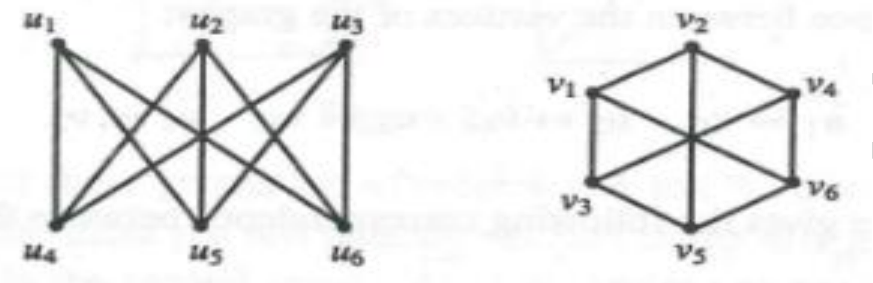
OR

2 a) Define an Euler trail and an Euler circuit. Show that a connected graph with exactly 2 vertices of odd degree has an Euler trail. **6**

b) Apply Dijkstra's algorithm to find the shortest path and its weight from vertex A to vertex Z in the weighted directed network shown below. **7**



c) Define Isomorphism of two graphs. Show that the following graphs are isomorphic: **7**



UNIT - II

3 a) Define Catalan number. In how many ways can one move from the point $(2,5)$ to the point $(8,11)$ in the xy-plane by using the moves $R:(x,y)\to(x+1,y)$ and $U:(x,y)\to(x,y+1)$ moves and without crossing the line $y=x+3$. **6**

b) Find the coefficient of $w^3x^2yz^2$ in the expansion of $(2w-x+3y-2z)^8$. **7**

c) In how many ways can the integers 1, 2, 3,…,10 be arranged in a line so that no even integer is in its natural place. **7**

UNIT - III

4 a) Derive the mean and standard deviation of Exponential distribution. **6**

b) Given that the probability of an accident in an industry is 0.005 and assuming the accidents are independent, **7**

i. Determine the probability that in any given period of 400 days, there will be an accident on a day?

ii. What is the probability that there are at most three accidents on a day?

c) A fair coin is tossed three times. Let X denote 0 or 1 depending on whether a head or a tail occurs on the first toss. Let Y denote the number of heads which occur. **7**

i. Find the marginal distributions of X and Y.

ii. Determine the joint distribution of X and Y.

iii. Determine the covariance of X and Y.

UNIT - IV

5 a) A company claims that the mean thermal efficiency of diesel engines produced by them is 32.3%. To test this claim, a random sample of 40 engines were examined which showed the mean thermal efficiency of 31.4% and standard deviation of 16%. Can this claim be accepted or not, at 0.01 level of significance? **6**

b) Under quality improvement programme, some teachers are trained by instruction methodology A and some by methodology B. In a random sample of size 10 taken from a large group of teachers exposed to each of these two methods, the following marks are obtained in an appropriate achievement test **7**

Method A	65	69	73	71	75	66	71	68	68	74
Method B	78	69	72	77	84	70	73	77	75	65

Assuming that the populations sampled are approximately normally distributed having the same variance, test the claim that method B is more effective at 0.05 level of significance.

c) The household net expenditure on health care in south and north India, in two sample of households, expressed as percentage of total income is shown the following table: **7**

North	15	8	3.8	6.4	27.4	19	35.5	13.6	–
South	18.8	23.1	10.3	8	18	10.2	15.2	19	20.2

Test the equality of variances of households' net expenditure on health care in south and north India at 5% level of significance.

OR

6 a) The following data gives the amount of androgen present in blood of 10 Deers before and 30 minutes after a certain drug is injected to them. **6**

Before	2.76	5.18	2.68	3.05	4.10	7.05	6.6	4.79	7.39	7.3
After	7.02	3.1	5.44	3.99	5.21	10.26	13.91	18.53	7.91	4.85

Test whether there is a significant change in the concentration levels of androgen in blood at 0.05 level of significance.

b) In a survey of AC machines produced by company **A** it was found that 19 machines were defective in a random sample of 200 while for company **B,** it is found that 5 were defective out of 100. At 5% level of significance, is there a reason to believe that the products of **B** are superior to products of **A**? **7**

c) The following data relates to the number of mistakes in each page of a book containing 180 pages. **7**

No. of mistakes per page	0	1	2	3	4	5 or more
No. of pages	123	31	10	5	6	5

Test whether Poisson distribution is a good fit to this observed distribution at 5% level of significance.

UNIT - V

7 a) An auto insurance company classifies its customers in three categories: poor, satisfactory and preferred. No one moves from poor to preferred or from preferred to poor in one year. 40% of the customers in the poor category become satisfactory, 30% of those in the satisfactory category moves to preferred, while 10% become poor; 20% of those in the preferred category are downgraded too satisfactory. **6**

i. Write the transition matrix for the model.

ii. Is the Markov chain irreducible?

b) A salesman's territory consists of 3 cities A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in city B. However, if he sells in either city B or C, then the next day he is twice as likely to sell in city A as in other city. **7**

i. Suppose that he sells in city B in the 1st week, find the probability of selling in the city C in the 3rd week.

ii. In the long run, how often does he sell in each of the cities?

c) The arrivals at a telephone booth are considered to be following Poisson law of distribution with an average time of 10 minutes between one arrival and the next. Length of the phone call is assumed to be distributed exponentially with a mean of 3 minutes. **7**

i) What is the probability that a person arriving at the booth will have to wait?

ii) What is the average length of queue that forms from time to time?

iii) The telephone department will install a second booth when convinced that an arrival would expect to wait at least three minutes for the phone. By how much must the flow of arrivals be increased in order to justify a second booth?

******