B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

August 2023 Semester End Make-Up Examinations

Programme: B.E.   Semester: III

Branch: CSE/ISE   Duration: 3 hrs.

Course Code: 22MA3BSSDM   Max Marks: 100

Course: Statistics and Discrete Mathematics   Date: 10.08.2023

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.
3. Use of Statistical tables is permitted.

UNIT - I

1 a) Let G be a graph of order 9 such that each vertex has degree 5 or 6. Prove that at least 5 vertices have degree 6 or at least 6 vertices have degree 5.   **6**

b) Draw a graph and write its adjacency matrix whose incidence matrix is given below.   **7**

$$\begin{array}{c c c c c c c c c} & a & b & c & d & e & f & g & h \\ v_1 & \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ v_2 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ v_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ v_4 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ v_5 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ v_6 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

c) Apply Kruskal's algorithm to find the minimal spanning tree and its weight for the weighted graph shown below: **7**
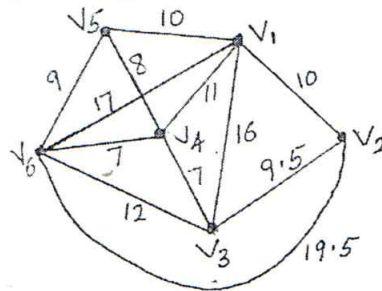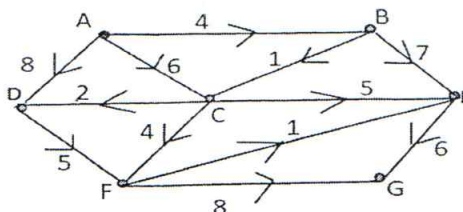

OR

2 a) Verify whether the two graphs shown below are isomorphic or not.   **6**
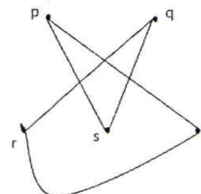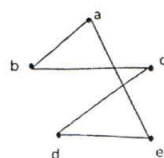
b) Let G be a disconnected graph of even order n with two components each of which is complete. Prove that G has a minimum of $n(n-2)/4$ edges. **7**

c) Apply the Dijkstra's algorithm to find the shortest path and its weight from vertex A to each of the vertices in the weighted directed network shown below. **7**

UNIT - II

3 a) Determine the coefficient of $x^2 y^2 z^3$ in the expansion of $(3x - 2y - 4z)^7$. **6**

b) Find the number of permutations of the letters $a, b, c, \ldots, x, y, z$ in which none of the patterns $spin, game, path$ or net occurs. **7**

c) At a restaurant, 10 men hand over their umbrellas to the receptionist. In how many ways can their umbrellas be returned so that **7**
(i) No man receives his own umbrella?
(ii) At least one of the men receives his own umbrella?
(iii) At least two of the men receives his own umbrella?

UNIT - III

4 a) Fit a second-degree parabola $y = ax^2 + bx + c$ in the least square sense for the following data: **6**

x	1	2	3	4	5
y	10	12	13	16	19

b) (i) Estimate the blood pressure (B.P.) of a woman of age 45 years by fitting a regression equation for the following data which shows the ages X and systolic B.P. (Y) of six women. **7**
(ii) Are the two variables ages (X) and B.P. (Y) correlated?

Age (X)	56	42	72	36	63	47
B.P. (Y)	147	125	160	118	149	128

c) In a certain factory turning out razor blades, there is a small probability of $1/500$ for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) no defective (ii) one defective (iii) two defective blades in a consignment of 10,000 packets. **7**

UNIT - IV

5 a) Mice with an average life span of 32 months will live up to 40 months when fed by a certain nutritious food. If 64 mice fed on this diet have an average lifespan of 38 months and standard deviation of 5.8 months, is there any reason to believe that average life span is less than 40 months. **6**

b) Under quality improvement programme some teachers are trained by instruction methodology A and some by methodology B. In a random sample of size 10, taken from a large group of teachers exposed to each of these two methods, the following marks are obtained in an appropriate achievement test.   **7**

| Method A | 65 | 69 | 73 | 71 | 75 | 66 | 71 | 68 | 68 | 74 |
| Method B | 78 | 69 | 72 | 77 | 84 | 70 | 73 | 77 | 75 | 65 |

Assuming that populations sampled are approximately normally distributed having same variance, test the claim that method B is more effective at 0.05 level of significance.

c) The household net expenditure on health care in south and north India, in two sample of households, expressed as percentage of total income is shown in the following table: **7**

| North | 15 | 8 | 3.8 | 6.4 | 27.4 | 19 | 35.5 | 13.6 | |
| South | 18.8 | 23.1 | 10.3 | 8 | 18 | 10.2 | 15.2 | 19 | 20.2 |

Test the equality of variances of households' net expenditure on health care in south and north India at 5% level of significance.

OR

6 a) An ambulance service company claims that on an average it takes 20 minutes between a call for an ambulance and the patient's arrival at the hospital. If in 6 calls the time taken are 27, 18, 26, 15, 20 and 32. Can the company's claim be accepted?   **6**

b) In a study of usefulness of yoga in weight reduction, a random sample of 7 persons undergoing yoga were examined of their weight before and after yoga with the following results: **7**

| Weight before | 209 | 178 | 169 | 212 | 180 | 192 | 158 |
| Weight after | 196 | 171 | 170 | 207 | 177 | 190 | 159 |

Test whether yoga is useful in weight reduction at 0.01 level of significance.

c) Test for goodness of fit of a poisson distribution at 0.05 level of significance to the following frequency distribution: **7**

| Number of patients arriving/hour: (x) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| Frequency | 52 | 151 | 130 | 102 | 45 | 12 | 5 |

UNIT - V

7 a) Solve the linear congruence $7x \equiv 2(\mod 37)$.   **6**

b) Find the smallest positive integer n such that $3|n+1$, $5|n+2$, $7|n+3$ and $11|n+4$.   **7**

c) Use the Fermat's Little theorem to verify that 17 divides $11^{104} + 1$.   **7**

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