U.S.N.										

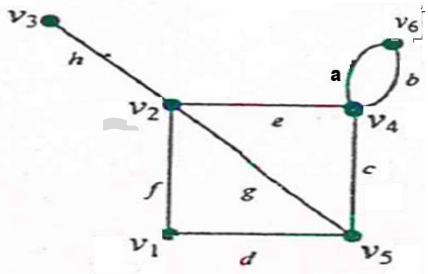
B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

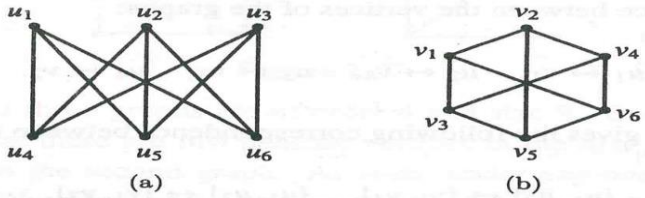
January / February 2025 Semester End Main Examinations

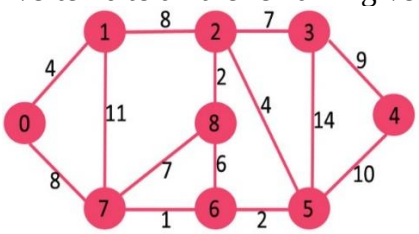
Programme: B.E. **Semester: III**
Branch: CSE/ISE/CS-DS/CS-IOT/AI-DS **Duration: 3 hrs.**
Course Code: 23MA3BSSDM **Max Marks: 100**
Course: STATISTICS AND DISCRETE MATHEMATICS

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.
3. Use of Statistical tables is permitted.

			UNIT -1	*CO*	*PO*	Marks
1	a)		Let G be a disconnected graph of even order n with two components each of which is complete. Prove that G has a minimum of $\frac{n(n-2)}{4}$ edges.	*1*	*1*	**6**
	b)		Write the incidence matrix of the graph given below with three observations on it. 	*1*	*1*	**7**
	c)		Eight cities A, B, C, D, E, F, G and H are required to be connected by a new railway network. The possible tracks and the cost involved to lay them (in crore of rupees) are summarized in the following table. Represent the network in terms of graph and hence apply Kruskal's algorithm to determine a railway network of minimal cost that connects all these cities.	*1*	*1*	**7**

Track	AB	AD	AG	BC	CD	CE	DF	EF	FG	FH	GH
Cost	155	145	120	145	150	95	100	150	140	150	160

			OR			
2	a)		Verify the following two graphs are isomorphic or not? 	*1*	*1*	**6**
	b)		Determine the order |V| of the graph $G = (V, E)$ in the following cases: i) G is a cubic graph with 9 edges. ii) G is a regular graph with 15 edges. iii) G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3.	*1*	*1*	**7**

		c)	Apply Dijkstra's algorithm to find the shortest path and its weight for the following graph from vertex 0 to all the remaining vertex. 	*1*	*1*	**7**
			UNIT - 2			
	3	a)	In a bookstore, on an average 5 customer visits per hour. Find the probabilities of the following scenarios: i) At a randomly selected hour, no customers visit. ii) At a randomly selected hour, exactly three customers visit. iii) At a randomly selected hour, at least 6 customers visit.	*1*	*1*	**6**
		b)	The kilometer run (in thousands of kms) without any sort of problem in respect of a certain vehicle is a variable having probability density function $f(x) = \begin{cases} \frac{1}{40} e^{-\frac{x}{40}}, x \geq 0 \\ 0, \quad x \leq 0 \end{cases}$. Find the probability that vehicle is trouble free for: i) At least for 25000 kms. ii) At most for 25000 kms. iii) Between 16000 to 32000 kms.	*1*	*1*	**7**
		c)	The total duration of baseball games in the major league in the 2011 season is uniformly distributed between 447 hours and 521 hours inclusive: i) Identify the points a and b and write its probability density function. ii) Find the mean and the standard deviation of the distribution. iii) What is the probability that the duration of games for a team in the 2011 season is between 480 and 500 hours?	*1*	*1*	**7**
			OR			
	4	a)	The probability that a student pilot passes the written test for a private pilot's license is 0.7. Find the probability that the student will pass the test i) On the third try. ii) Before the fourth try.	*1*	*1*	**6**
		b)	The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be i) Less than 65 ii) More than 75 iii) Between 65 and 75.	*1*	*1*	**7**
		c)	A repair shop observes that the time to complete repairs for a certain type of machine follows a Gamma distribution with a shape parameter $\alpha = 2$ and a rate parameter $\beta = 0.5$. i) What is the probability that the repair time will take less than 3 hours? ii) Find the mean and variance of the repair time distribution.	*1*	*1*	**7**

			UNIT – 3			
	5	a)	Suppose X and Y are independent discrete random variables with the following respective distribution. Find the joint distribution of X and Y. Also verify $COV(X,Y) = 0$. x_i | 1 | 2 $f(x_i)$ | 0.7 | 0.3 y_j | -2 | 5 | 8 $g(y_j)$ | 0.3 | 0.5 | 0.2	*1*	*1*	**6**
		b)	Find the value of the constant $'c'$ such that $f(x,y)$ is a joint probability density function of two continuous random variables X and Y. $f(x,y) = \begin{cases} c(2x+y), 0 \le x \le 2, 0 \le y \le 3 \\ 0 \ , otherwise \end{cases}$ Hence evaluate $P(x \ge 1, y \le 2)$.	*1*	*1*	**7**
		c)	A student's study habits are as follows. If he studies one night, he is 70% sure not to study the next night. On the other hand, if he does not study one night, he is 60% sure not to study the next night. Write the transition matrix and hence find in the long run, how often does he study?	*1*	*1*	**7**
			OR			
	6	a)	The joint probability distribution of two discrete random variables X and Y is given by $f(x,y) = k(2x+y)$ where x and y are integers such that $0 \le x \le 2, 0 \le y \le 3$. i) Find the value of the constant k. ii) Find the marginal probability distributions of X and Y. iii)Show that the random variables X and Y are dependent.	*1*	*1*	**6**
		b)	The joint density function of two continuous random variables X and Y is given by $f(x,y) = \begin{cases} \frac{xy}{96}; & 0 < x < 4, 1 < y < 5 \\ 0; & otherwise \end{cases}$. Find $P(X+Y<3)$.	*1*	*1*	**7**
		c)	Three boys A, B, C are throwing ball to each other. The boy A always throws the ball to boy B and boy B always throws the ball to boy C. Boy C is just as likely to throw the ball to boy B as to boy A. If boy C was the first person to throw the ball, find the probabilities that after three throws i) Boy A has the ball ii) Boy B has the ball iii) Boy C has the ball.	*1*	*1*	**7**
			UNIT - 4			
	7	a)	Mice with an average lifespan of 32 months will live up to 40 months when fed by a certain nutritious food. If 64 mice fed on this diet have an average lifespan of 38 months and standard deviation of 5.8 months, is there any reason to believe that the average lifespan is less than 40 months at $\alpha = 5\%$.	*1*	*1*	**6**
		b)	Out of random sample of 9 mice suffering with a disease, 5 mice were treated with a new serum while the remaining were not treated. From the time of commencement of experiment, the following are the survival times: Treatment | 2.1 | 5.3 | 1.4 | 4.6 | 0.9 No Treatment | 1.9 | 0.5 | 2.8 | 3.1 | _- Test whether the serum treatment is effective in curing the disease at 5% level of significance, assuming that the two distributions are normally distributed with equal variances.	*1*	*1*	**7**

	c)	It is desired to test whether the number of gamma rays emitted per second by a certain radioactive substance is a random variable having the Poisson distribution with mean 2.4. Use the following data obtained for 300 one-second intervals to test this null hypothesis at 5% level of significance. No. of gamma rays: 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 or more Frequency: 18 | 48 | 66 | 74 | 44 | 35 | 10 | 5	*1*	*1*	**7**
		OR				
8	a)	The average weekly losses of man-hours due to strikes in an institute before and after a disciplinary program was implemented are as follows: Is there a reason to believe that the disciplinary program is effective at 5% level of significance? Before: 45 | 73 | 46 | 124 | 33 | 57 | 83 | 34 | 26 | 17 After: 36 | 60 | 44 | 119 | 35 | 51 | 77 | 29 | 24 | 11	*1*	*1*	**6**
	b)	If a random sample data show that 42 men earn on the average $\bar{x}_1 = 744.85$ with standard deviation $s_1 = 397.7$. While 32 women earn on the average $\bar{x}_2 = 516.78$ with standard deviation $s_2 = 162.523$. Test at 0.05 level of significance whether the average income of men exceeds that of women.	*1*	*1*	**7**
	c)	The household net expenditure on health care in south and north India, in two samples of households, expressed as percentage of total income is shown in the following table: South: 15 | 8 | 3.8 | 6.4 | 27.4 | 19 | 35.3 | 13.6 | - North: 18.8 | 23.1 | 10.3 | 8 | 18 | 10.2 | 15.2 | 19 | 20.2 Test the equality of variances of household's net expenditure on health care in south and north India at 1% level of significance.	*1*	*1*	**7**
		UNIT - 5				
9	a)	Define Catalan number. Obtain the number of paths from $(2,1)$ to $(7,6)$ and not rise above the line $y = x-1$ using the moves $R: (x, y) \to (x+1, y)$ and $U: (x, y) \to (x, y+1)$.	*1*	*1*	**6**
	b)	In how many ways 5 number of a's, 4 number of b's and 3 number of c's can be arranged so that all the identical letters are not in a single block?	*1*	*1*	**7**
	c)	At a restaurant, 10 men hand over their umbrellas to the receptionist. In how many ways can their umbrellas be returned so that **i)** No man receives his own umbrella? **ii)** At least one of the men receives his own umbrella? **iii)** At most two of the men receives his own umbrella?	*1*	*1*	**7**
		OR				
10	a)	Define derangement and hence obtain d_5, d_6, d_7, d_8.	*1*	*1*	**6**
	b)	In how many ways can the 26 letters of the English alphabet be permuted so that none of the pattern's CAR, DOG, PUN or BYTE occurs?	*1*	*1*	**7**
	c)	Determine the coefficient of i) x^9y^3 in the expansion of $(2x-3y)^{12}$. ii) $a^2b^3c^2d^5$ in the expansion of $(a+2b-3c+2d+5)^{16}$.	*1*	*1*	**7**

****