B.M.S. COLLEGE OF ENGINEERING, BENGALURU-19
(*Autonomous Institute, Affiliated to VTU***)**
DEPARTMENT OF MATHEMATICS
THIRD SEMESTER B.E COURSE(CSE/ISE/CSE-AIDS/CS-IOT/CSE-DS)

Course Title: Statistics and Discrete Mathematics
Course Code: 23MA3BSSDM

UNIT 4: STATISTICAL INFERENCE

I. Test of significance for single mean [large sample: $n \geq 30$]

Test statistic	Null Hypothesis	Alternative Hypothesis	Accept H_o
$Z = \dfrac{\bar{x} - \mu_O}{\sigma / \sqrt{n}}$	$H_0 : \mu = \mu_O$	$H_1 : \mu \neq \mu_O$	$\lvert z \rvert < z_{\alpha/2}$
		$H_1 : \mu < \mu_O$	$z > z_\alpha$
		$H_1 : \mu > \mu_O$	$z < z_\alpha$

1. The length of life X of certain computers is approximately normally distributed with mean 800 hours and S.D 40 hours. If a random sample of 30 computers has an average life of 788 hours, test the null hypothesis that μ=800 hours at (a) 5% (b) 1% (c) 10% (d) 15% level of significance
2. Mice with an average lifespan of 32 months will live upto 40 months when fed by a certain nutrious food. If 64 mice fed on this diet have an average lifespan of 38 months and standard deviation of 5.8 months, is there any reason to believe that average lifespan is less than 40 months.
3. A machine runs on an average of 125 hours/year. A random sample of 49 machines has an annual average use of 126.9 hours with standard deviation 8.4 hours. Does this suggest to believe that machines are used on the average more than 125 hours annually at 0.05 level of significance?
4. A company claims that the mean thermal efficiency of diesel engines produced by them is 32.3%. To test this claim, a random sample of 40 engines were examined which showed the mean thermal efficiency of 31.4% and S.D of 1.6%. Can the claim be accepted or not, at 0.01 L.O.S.?
5. It has previously been recorded that the average depth of ocean at a particular region is 67.4 fathoms. Is there reason to believe this at 0.01 L.O.S. if the readings at 40 random locations in that particular region showed a mean of 69.3 with S.D of 5.4 fathoms?
6. A company producing computers states that the mean lifetime of its computers is 1600 hours. Test this claim at 0.01 L.O.S. against the A.H.: μ < 1600 hours if 100 computers produced by this company has mean lifetime of 1570 hours with S.D. of 120 hours.
7. A manufacturer of tyres guarantees that the average lifetime of its tyres is more than 28000 miles. If 40 tyres of this company tested, yields a mean lifetime of 27463 miles with S.D. of 1348 miles, can the guarantee be accepted at 0.01 L.O.S.?

II. Test of significance for difference between two means [large sample: $n \geq 30$]

Test statistic	Null Hypothesis	Alternative Hypothesis	Accept H_o
$Z = \dfrac{(\bar{x}_1 - \bar{x}_2) - \delta}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$	$H_0 : \mu_1 - \mu_2 = \delta$	$H_1 : \mu_1 - \mu_2 \neq \delta$	$\lvert z \rvert < z_{\alpha/2}$
		$H_1 : \mu_1 - \mu_2 > \delta$	$z < z_\alpha$
		$H_1 : \mu_1 - \mu_2 < \delta$	$z > z_\alpha$

1. In a random sample of 100 tube lights produced by company A, the mean lifetime (mlt) of tube light is 1190 hours with standard deviation of 90 hours. Also in a random sample of 75 tube lights from company B the mean lifetime is 1230 hours with standard deviation of 120 hours. Is there a difference between the mean lifetimes of the two brands of tube lights at a significance level of (a) 0.05 (b) 0.01?
2. To test the effects a new pesticide on rice production, a farm land was divided into 60 units of equal areas, all portions having identical qualities as to soil, exposure to sunlight etc. The new pesticide is applied to 30 units while old pesticide to the remaining 30. Is there reason to belive that the new pesticide is better than the old pesticide if the mean number of kgs of rice harvested/units using new pesticide (N.P.) is 496.31 with S.D. of 17.18 kgs. Test at a level of significance (a) 0.05 (b) 0.01?
3. A random sample of 40 ‘geysers’ produced by company A have a mean lifetime (mlt) of 647 hours of continuous use with a S.D. of 27 hours, while a sample 40 produced by another company B have mlt of 638 hours with S.D. 31 hours. Does this substantiate the claim of company A that their ‘geyers’ are superior to those produced by company B at (a) 0.05 (b) 0.01 L.O.S.
4. Test the N.H.: $\mu_A - \mu_B = 0$ against the A.H.: $\mu_A - \mu_B \neq 0$ at 0.01 L.O.S. for the following data

	Sample Size	Mts (kgs)	S.D. (kgs)
Type A	40	247.3	15.2
Type B	30	254.1	18.7

5. If a random sample data show that 42 men earn on the average $\bar{x}_1 = 744.85$ with S.D. $s_1 = 397.7$ while 32 women earn on the average $\bar{x}_2 = 516.78$ with S.D. $s_2 = 162.523$, test at 0.05 level of significance whether the average income for men and women is same or not.

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III. Small Sample Test Concerning Single Mean: t-Distribution

Test statistic	Null Hypothesis	Alternative Hypothesis	Accept H_o
$t=\dfrac{\bar{x}-\mu_O}{S/\sqrt{n}}$ here $S^2=\dfrac{\sum(x_i-\bar{x})^2}{n-1}$ **OR** $t=\dfrac{\bar{x}-\mu_O}{s/\sqrt{n-1}}$ here $s^2=\dfrac{\sum(x_i-\bar{x})^2}{n}$	$H_0:\mu=\mu_O$	$H_1:\mu\neq\mu_O$	$\lvert t\rvert<t_{\alpha/2,n-1}$
		$H_1:\mu<\mu_O$	$t>t_\alpha$
		$H_1:\mu>\mu_O$	$t<t_\alpha$

1. An ambulance service company claims that on an average it takes 20 minutes between a call for an ambulance and the patient's arrival at the hospital. If in 6 calls the time taken (between a call and arrival at hospital) are 27, 18, 26, 15, 20 and 32. Can the company's claim be accepted?
2. Mean lifetime of computers manufactured by a company is 1120 hours with standard deviation of 125 hours. (a) Test the typothesis that mean lifetime of computers has not changed if a sample of 8 computers has a mean life time of 1070 hours (b) Is there decrease in mlt? Use (i) 5% (ii) 1% L.O.S.
3. An auditor claims that he takes on an average 10.5 days to file income tax returns (I.T. returns). Can this claim be accepted if a random sample shows that he took 13, 19, 15, 10, 12, 11, 14, 18 days to file I.T. returns? Use (a) 0.01 (b) 0.05 L.O.S.
4. If 5 pieces of certain ribbon selected at random have mean breaking strength of 169.5 pounds with S.D. of 5.7, do they confirm to the specification mean breaking strength of 180 pounds.
5. In a random sample of 10 bolts produced by a machine the mean length of bolt is 0.53 mm and S.D 0.03 mm. Can we claim from this that the machine is in proper working order if in the past it produced bolts of length 0.50 mm? Use (a) 0.05 (b) 0.01 L.O.S.

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IV. Small Sample Test Concerning Difference Between Two Means: t-Distribution

<table>
<tr><th>Test statistic</th><th>Null Hypothesis</th><th>Alternative Hypothesis</th><th>Accept H_o</th></tr>
<tr><td rowspan="3">$$t=\frac{(\bar{x}_1-\bar{x}_2)-\delta}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$
(i) $S^2=\frac{\sum(x_{1i}-\bar{x}_1)^2+\sum(x_{2i}-\bar{x}_2)^2}{n_1+n_2-2}$
OR
(ii) $S^2=\frac{n_1s_1^2+n_2s_2^2}{n_1+n_2-2}$
if $s_1^2=\frac{\sum(x_{1i}-\bar{x}_1)^2}{n_1}$ and $s_2^2=\frac{\sum(x_{2i}-\bar{x}_2)^2}{n_2}$
OR
(iii) $S^2=\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}$
if $S_1^2=\frac{\sum(x_{1i}-\bar{x}_1)^2}{n_1-1}$ and $S_2^2=\frac{\sum(x_{2i}-\bar{x}_2)^2}{n_2-1}$</td><td rowspan="3">$H_0:\mu_1-\mu_2=\delta$</td><td>$H_1:\mu_1-\mu_2\neq\delta$</td><td>$|t|<t_{\alpha/2},_{n_1+n_2-2}$</td></tr>
<tr><td>$H_1:\mu_1-\mu_2>\delta$</td><td>$t<t_\alpha$</td></tr>
<tr><td>$H_1:\mu_1-\mu_2<\delta$</td><td>$t>t_\alpha$</td></tr>
</table>

1. In a mathematics examination 9 students of class A and 6 students of class B obtained the following marks. Test at 1% L.O.S. whether the performance in mathematics is same or not for the two classes A and B. assume that the samples are drawn from normal populations having same variance.

A:	44	71	63	59	68	46	69	54	48
B:	52	70	41	62	36	50	-	-	-

2. Out of random sample of 9 mice, suffering with a disease, 5 mice were treated with a new serum while the remaining were not treated. From the time commencement of experiment, the following are the survival times:

Treatment	2.1	5.3	1.4	4.6	0.9
NoTreatment	1.9	0.5	2.8	3.1	-

Test whether the serum treatment is effective in curing the disease at 5% L.O.S., assuming that the two distributions are normally distributed with equal variances.

3. Random samples of specimens of coal from two mines A and B are drawn and their heat producing capacity (in millions of calories/ton) were measured yielding the following results:

Mine A:	8350	8070	8340	8130	8260	-
Mine B:	7900	8140	7920	7840	7890	7950

Is there significant difference between the means of these two samples at 1% L.O.S.?

4. To determine whether vegetarian and non-vegetarian diets effects significantly on increase in weight a study was conducted yielding the following data of gain in weight.

Veg	34	24	14	32	25	32	30	24	30	31	35	25	-	-	-
Non-Veg	22	10	47	31	44	34	22	40	30	32	35	18	21	35	29

Can we claim that the two diets differ pertaining to weight gain, assuming that samples are drawn from normal populations with same variance?

5. A group of 10 rats fed on a diet A and another group of 8 rats fed on a different diet B recorded the following increase in weights.

Diet A	5	6	8	1	12	4	3	9	6	10
Diet B	2	3	6	8	10	1	2	8	-	-

Does it show the superiority of diet A over that of B?

6. A group of 5 patients treated with medicine "A" weigh 42, 39, 48, 60, and 41 kgs. A second group of 7 patients from the same hospital treated with medicine "B" weigh 38, 42, 56, 64, 68, 69 and 62 kgs. Do you agree with the claim that the medicine "B" increases the weight significantly?

V. **Paired Sample t-Test**

Test statistic	Null Hypothesis	Alternative Hypothesis	Accept H_o
$t=\dfrac{\bar{d}-\mu_d}{S_d/\sqrt{n}}$ here $S_d^2=\dfrac{\sum\left(d_i-\bar{d}\right)^2}{n-1}$ OR $t=\dfrac{\bar{d}-\mu_d}{S_d/\sqrt{n-1}}$ here $S_d^2=\dfrac{\sum\left(d_i-\bar{d}\right)^2}{n}$	$H_0:\mu=\mu_d$	$H_1:\mu\neq\mu_d$	$\lvert t\rvert<t_{\alpha/2,n-1}$
		$H_1:\mu<\mu_d$	$t>t_{\alpha}$
		$H_1:\mu>\mu_d$	$t<t_{\alpha}$

1. Use paired sample test at 0.05 L.O.S. to test from the following data whether the differences of means of the weights obtained by two different scales (weighting machines) is significant.

Scale: I	11.23	14.36	8.33	10.50	23.42	9.15	13.47	6.47	12.40	19.38
Scale: II	11.27	14.41	8.35	10.52	23.41	9.17	13.52	6.46	12.45	19.35

2. The average weekly losses of man-hours due to strikes in an institute before and after a disciplinary program was implemented are as follows: Is there reason to believe that the disciplinary program is effective at 5% L.O.S.?

Before	45	73	46	124	33	57	83	34	26	17
After	36	60	44	119	35	51	77	29	24	11

3. The pulsality index (P.I.) of 11 patients before and after contracting a disease are given below. Test at 0.05 L.O.S. whether there is a significant increase of the mean of P.I. values.

Before	0.4	0.45	0.44	0.54	0.48	0.62	0.48	0.60	0.45	0.46	0.35
After	0.5	0.60	0.57	0.65	0.63	0.78	0.63	0.80	0.69	0.62	0.68

4. The blood pressure (B.P.) of 5 women before and after intake of a certain drug are given below:

Before	110	120	125	132	125
After	120	118	125	136	121

Test at 1% L.O.S. whether there is significant change in B.P.

5. Marks obtained in mathematics by 11 students before and after intensive coaching are given below:

Before	24	17	18	20	19	23	16	18	21	20	19
After	24	20	22	20	17	24	20	20	18	19	22

Test at 5% L.O.S. whether the intensive coaching is useful?

VI. Ratio of Variances: F-Distributions

Test statistic	Null Hypothesis	Alternative Hypothesis	Accept H_o
$F = \dfrac{S_1^2}{S_2^2}$ if $S_1^2 > S_2^2$ OR $F = \dfrac{S_2^2}{S_1^2}$ if $S_2^2 > S_1^2$ here $S_1^2 = \dfrac{n_1 s_1^2}{n_1 - 1} = \dfrac{\sum\left(x_{1i} - \bar{x}_1\right)^2}{n_1 - 1}$; $S_2^2 = \dfrac{n_2 s_2^2}{n_2 - 1} = \dfrac{\sum\left(x_{2i} - \bar{x}_2\right)^2}{n_2 - 1}$	$H_0 : \sigma_1^2 = \sigma_2^2$	$H_1 : \sigma_1^2 \neq \sigma_2^2$	$F < F_{n_1 - 1, n_2 - 1, \alpha}$ OR $F < F_{n_2 - 1, n_1 - 1, \alpha}$

1. The household net expenditure on health care in south and north India, in two samples of households, expressed as percentage of total income is shown the following table:

South	15	8	3.8	6.4	27.4	19	35.3	13.6	
North	18.8	23.1	10.3	8	18	10.2	15.2	19	20.2

   Test the equality of variances of household's net expenditure on health care in south and north India.

2. Can we conclude that the two population variances are equal for the following data of post graduates passed out from a state and private university.

State:	8350	8260	8130	8340	8070	
Private:	7890	8140	7900	7950	7840	7920

3. Is there reason to believe that the life expected in south and north India is same of not from the following data

South:	34	39.2	46.1	48.7	49.4	45.9	55.3	42.7	43.7		
North:	49.7	55.4	57	54.2	50.4	44.2	53.4	57.5	61.9	56.6	58.2

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VII. Chi-Square Distribution: Goodness of Fit

Test statistic	**Null Hypothesis**	**Alternative Hypothesis**	**Accept H_o**
$\chi^2 = \frac{\sum_{i=1}^{n}(o_i - e_i)^2}{e_i}$	H_0 : There is no significant difference between experimental and theoretical values	H_1 : There is significant difference between experimental and theoretical values	$\chi^2 < \chi^2_{n-k,\alpha}$

1) A sample analysis of examination results of 500 students was made. It was found that 220 students had failed, 170 had secured third class, 90 had secured second class and 20 had secured first class. Do these figures support the general examination result which is in the ratio 4:3:2:1 for the respective categories?

2) In a Mendelian experiment on breeding, four types of plants are expected to occur in the proportion 9:3:3:1. The observed frequencies are 891 round and yellow, 316 wrinkled and yellow, 290 round and green, and 119 wrinkled and green. Find the Chi-square value and examine the correspondence between the theory and the experiment.

3) A machine is supposed to mix peanuts, hazelnuts, cashews and pecans in the ratio 5:2:2:1. A can containing 500 of these mixed nuts was found to have 269 peanuts, 112 hazel nuts, 74 cashews and 45 pecans. Can we conclude that the machine is mixing the nuts in the stated ratio?

4) It is believed that the proportion of people with A, B, O and AB blood types in a population are respectively 0.4, 0.2 0.3 and 0.1. When 400 randomly picked people were examined, the number of persons with these types was observed to be 148, 96,106 and 50 respectively. Test the hypothesis that these data bear out the stated belief.

5) Genetic theory states that children having one parent of blood type M and other blood type N will always are one of three types M, MN, N and that the proportions of these types will on average be 1: 2: 1. A report states that out of 300 children having one parent M and one N parent 30% were found to be of type M, 45% of type MN and remainder of type N. Find the Chi-square value and examine the correspondence between the theory and the experiment.

6) Among64 off-springs of a certain cross between pigs, 34 were red, 10 were black and 20 were white. According to a genetic model, these numbers should be in proportions 9:3:4. Is the data consistent with the model at 5% level?

7) In 1000 extensive sets of trials for an event of small probability, the frequency fo of the number x of seeds proved to be

x	0	1	2	3	4	5	6	7
f	304	366	210	80	28	9	2	1

Fit a Poisson distribution to the data and test the goodness of fit.

8) The number of cars passing a given point in 100 five second interval was observed as follows

No of Cars	0	1	2	3	4	5
No of intervals	40	30	14	6	5	5

Fit a Poisson distribution and test for its goodness of fit.

9) Test for goodness of fit of a Poisson distribution at 5% L.O.S. to the following frequency distribution:

No. of patients arriving/hour: (x)	0	1	2	3	4	5	6	7	8
Frequency	52	151	130	102	45	12	5	1	2

10) It is desired to test whether the number of gamma rays emitted per second by a certain radioactive substance is a random variable having the Poisson distribution with mean 2.4. Use the following data obtained for 300 one-second intervals to test this null hypothesis.

Number of gamma rays	0	1	2	3	4	5	6	7 or more
Frequency	18	48	66	74	44	35	10	5

UNIT-4
STATISTICAL INFERENCE

★ **Components of a Hypothesis Test:-**

- Step 1: Formulate the hypothesis to be tested
- Step 2: Determine the appropriate test statistic and calculate it using sample data
- Step 3: Comparison of test statistics to critical region
- Step 4: Draw a conclusion.

NOTE: Normal ← $n \geq 30$ large sample

- t-test ← $n < 30$ small sample
- Type I error → α
- Type II error → β

★ TYPE 1:-

★ Single mean test (for large sample):

Test statistic	Null Hypothesis	Alternative Hypothesis	Accept H_0
$Z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$H_0 : \mu = \mu_0$	$H_1 : \mu \neq \mu_0$	$\lvert z \rvert < Z_{\alpha/2}$
		$H_1 : \mu < \mu_0$	$Z > Z_\alpha$
		$H_1 : \mu > \mu_0$	$Z < Z_\alpha$

★ Problems:

..ans $x \rightarrow$ life of the electric bulb

$\mu = 360$; $\sigma = 90$; $n = 625$; $\bar{x} = 355$, $s = 90$

$H_0 : \mu = 360$ [sample is drawn from the population]

$H_1 : \mu \neq 360$ [it is not drawn from the population]

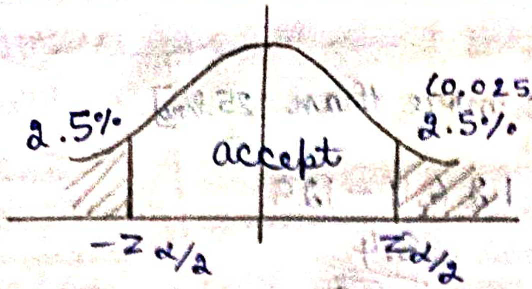
Step 2:

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{355 - 360}{\frac{90}{\sqrt{625}}} = \frac{-25}{18} = -1.3889$$

At $\alpha = 5\%$,

Decision making: Accept H_0 if $\lvert z \rvert < Z_{\alpha/2}$

we have taken $H_1 : \mu \neq 360$ it can be less or greater.



$0.5 - \phi(z_{\alpha/2}) = 0.025$

$\phi(z_{\alpha/2}) = 0.475$

$\therefore z_{\alpha/2} = 1.96$

Conclusion: $|z| = 1.3889$

as $|z| = 1.3889 < z_{\alpha/2} = 1.96$

$\therefore$ Accept H_0

2.ans $x \to$ Grade of a student

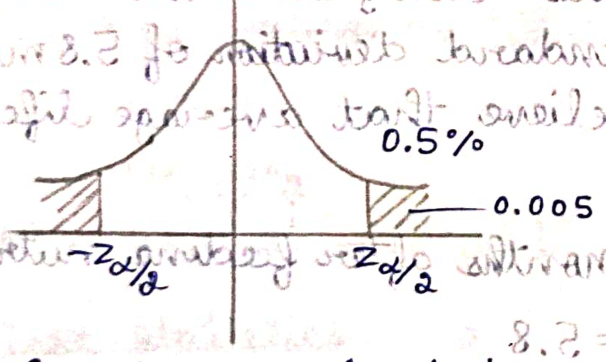
$\mu = 74.5$, $\sigma = 8$, $n = 200$, $\bar{x} = 75.9$, $\alpha = 1\%$

$H_0 : \mu = 74.5$

$H_1 : \mu \neq 74.5$

Test statistic : $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{75.9 - 74.5}{8/\sqrt{200}} = 2.4748$

Decision making: At $\alpha = 1\%$, accept H_0 if $|z| < z_{\alpha/2}$



$0.5 - \phi(z_{\alpha/2}) = 0.005$

$\phi(z_{\alpha/2}) = 0.495$

$\therefore z_{\alpha/2} = 2.57$

Conclusion: As $|z| = 2.4748 < z_{\alpha/2} = 2.57$

$\therefore$ Accept H_0.

3. A machine runs on an average of 125 hours/year. A random sample of 49 machines has an annual average use of 126.9 hours with standard deviation 8.4 hours. Does this suggest to believe that machines are used on the average more than 125 hours annually at 0.05 level of significance?

ans $x \to$ Machine run time in hrs/year.

$\mu = 125$, $n = 49$, $\bar{x} = 126.9$, $s = 8.4$, $\alpha = 0.05$

$\sigma = 8.4 = s$

$H_0: \mu \leq 125$

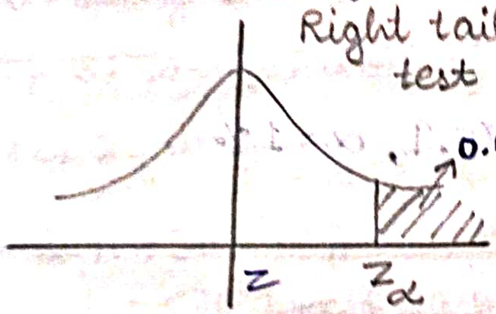
$H_1: \mu > 125$ [Machine runs for more than 125 hours]

$\mu > 125$
complement:
$\mu \leq 125$
whichever has (=) equal to sign, that is H_0.

Test statistics: $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{126.9 - 125}{8.4/\sqrt{49}}$

$z = \underline{\underline{1.5833}}$

Decision making: At $\alpha = 5\%$, accept H_0 if $Z < Z_\alpha$



$0.5 - \phi(z_\alpha) = 0.05$

$\phi(z_\alpha) = 0.45$

$z_\alpha = \underline{\underline{1.65}}$

Conclusion: As $z = 1.5833 < z_\alpha = 1.65$, $\therefore$ Accept H_0.

4. Mice with an average lifespan of 32 months will live upto 40 months when fed by a certain nutricious food. If 64 mice fed on this diet, have an average lifespan of 38 months & standard deviation of 5.8 months, is there any reason to believe that average lifespan is less than 40 months.

ans $x \to$ lifespan of a mice in months after feeding nutricious diet.

$\mu = 40$, $n = 64$, $\bar{x} = 38$, $s = 5.8$

$H_0: \mu \geq 40$

$H_1: \mu < 40$

Take upto 4 decimal places

Test statistics: $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{38 - 40}{5.8/\sqrt{64}} = -2.7586$

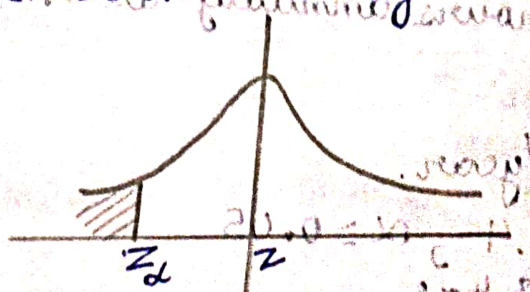
when α is not given, take $\alpha = 1\%$ or 5%

Decision making: At $\alpha = 1\%$, accept H_0 if $z > z_\alpha$

$0.5 - \phi(z_\alpha) = 0.01$

$\phi(z_\alpha) = 0.49$

$z_\alpha = \underline{\underline{-2.33}}$



Conclusion: At $\alpha = 1\%$, as $z = -2.7586 \not> -2.33$

$\therefore$ Reject H_0

TYPE-2: (Difference of mean for large sample) Two populations & one sample in each

★ Two Mean:

Test statistics: $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$: accept H_0 if $|z| < z_{\alpha/2}$

$H_1 : \mu_1 < \mu_2$ accept H_0 if $z > z_\alpha$

$H_1 : \mu_1 > \mu_2$ accept H_0 if $z < z_\alpha$

★ Problems:

Difference between the two means:

1.ans $x_1, x_2 \rightarrow$ wages of workers in A, B

$n_1 = 300$, $\bar{x}_1 = 1500$, $\sigma_1 = 500$; $n_2 = 325$, $\bar{x}_2 = 1550$, $\sigma_2 = 510$

$\alpha = 5\%$

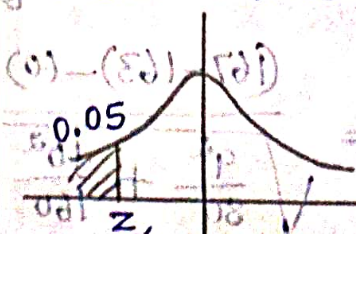
$H_0 : \mu_1 \geq \mu_2$

$H_1 : \mu_1 < \mu_2$ [Avg. wage in B is higher than wage in A]

• Test statistics: $Z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} = \dfrac{(1500 - 1550) - (0)}{\sqrt{\dfrac{(500)^2}{300} + \dfrac{(510)^2}{325}}}$

$$z = -1.2370$$

• Decision rule: At $\alpha = 5\%$ accept H_0 if $z > z_\alpha$



$0.5 - \phi(z_\alpha) = 0.05$

$\phi(z_\alpha) = 0.45$

$z_\alpha = -1.65$

- Conclusion: At $\alpha = 5\%$ $z = -1.237 > z_\alpha = -1.65$, accept H_0

$\therefore$ [The claim is wrong] write the claim if not mentioned earlier

2. ans $x_1, x_2 \rightarrow$ no. of mangoes stored in closed & open godown

$n_1 = 1000$, $n_2 = 800$, $\bar{x}_1 = 210$, $\bar{x}_2 = 200$, $\sigma_1 = 45$, $\sigma_2 = 42$

no condition specified, so take $\mu_1 \neq \mu_2$.

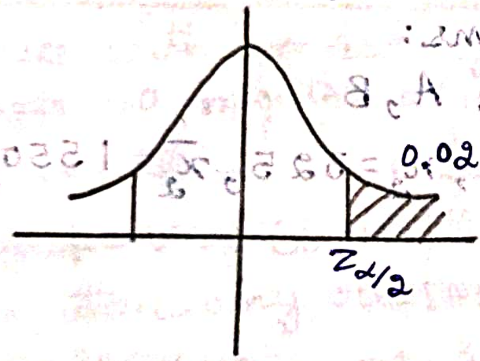
$H_0 : \mu_1 = \mu_2$ [not affected by weather condition]

$H_1 : \mu_1 \neq \mu_2$ [affected by weather condition]

Step 2: Test statistics: $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} = \dfrac{(210-200)-(0)}{\sqrt{\dfrac{(45)^2}{1000} + \dfrac{(42)^2}{800}}}$

$= 4.8621$

Decision rule: At $\alpha = 5\%$ accept H_0 if $|z| < z_{\alpha/2}$



$0.5 - \phi(z_{\alpha/2}) = 0.025$

$\phi(z_{\alpha/2}) = 0.475$

$z_{\alpha/2} = \underline{1.96}$

Conclusion: At $\alpha = 5\%$ as $|z| = 4.8621 \not< z_{\alpha/2} = 1.96$

$\therefore$ Reject H_0.

3. ans $x_1, x_2 \rightarrow$ Mean height of boys who participated & who did not participate respectively in cm

$n_1 = 80$, $n_2 = 160$, $\bar{x}_1 = 167$, $\bar{x}_2 = 163$, $\sigma_1 = 9$, $\sigma_2 = 10$, $\alpha = 5\%$

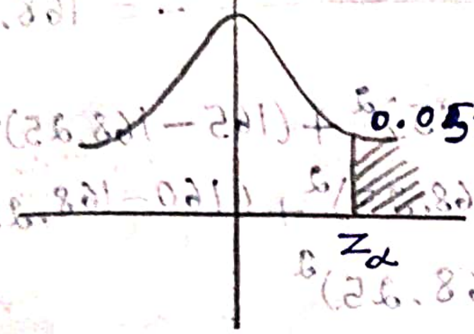
$H_0 : \mu_1 \leq \mu_2$

$H_1 : \mu_1 > \mu_2$ [Students participating in athletics are taller than others]

Test statistics: $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} = \dfrac{(167-163)-(0)}{\sqrt{\dfrac{9^2}{80} + \dfrac{10^2}{160}}}$

$z = 3.1258$

Decision rule: At $\alpha = 5\%$ accept H_0 if $z < z_\alpha$



$$0.5 - \phi(z_\alpha) = 0.05$$
$$\phi(z_\alpha) = 0.45$$
$$z_\alpha = 1.65$$

Conclusion: At $\alpha = 5\%$, $z = 3.1258 > z_\alpha = 1.65$, (i.e. $z \nless z_\alpha$)

$\therefore$ Reject H_0.

☆ TYPE-3: Single Mean for Small Sample

When the sample size is small it does not follow normal distribution but follows t-distribution. The curve of t-distribution is also symmetric about $\mu = 0$ & it has long, fat tail.

$S \to$ SD of population
$s \to$ SD of sample

Test statistics: $t = \dfrac{\bar{x} - \mu}{S/\sqrt{n}}$ (or) $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n-1}}$

where $S^2 = \dfrac{1}{n-1} \Sigma (x_i - \bar{x})^2$

$s^2 = \dfrac{1}{n} \Sigma (x_i - \bar{x})^2$ In calci, $\sigma x = S$

also $S^2 = \dfrac{n}{n-1} s^2$ only for calculator

where S is sample S.D. $S^2 \to$ variance

☆ Problems:

Single mean for small sample.

1.ans $x \to$ height of a male student

$x_i = 175, 168, 165, 170, 167, 160, 173, 168$

$n = 8$, $\alpha = 5\%$, $\mu = 165$

$H_0: \mu \leq 165$

$H_1: \mu > 165$ [mean height is greater than 165]

Test statistics: $t = \dfrac{\bar{x} - \mu}{S/\sqrt{n}}$, $S^2 = \dfrac{\Sigma (x_i - \bar{x})^2}{n-1}$

Mean is fixed, others can be changed. ∴ Degree of freedom = $n-1$

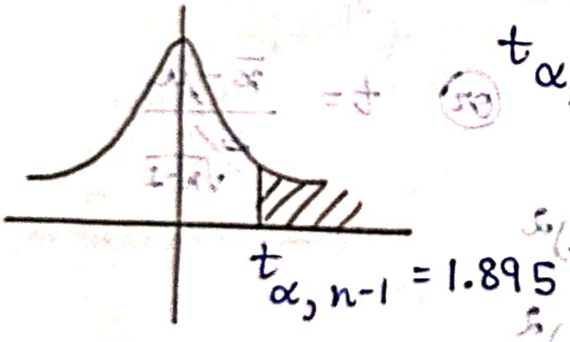
$$\bar{x} = \frac{175+168+165+170+167+160+173+168}{8} = 168.25$$

$$S^2 = \frac{(175-168.25)^2+(168-168.25)^2+(165-168.25)^2+(170-168.25)^2+(167-168.25)^2+(160-168.25)^2+(173-168.25)^2+(168-168.25)^2}{7}$$

$S^2 = 21.6428$; $S = 4.6521$

$$t = \frac{168.25-165}{\frac{4.6521}{\sqrt{8}}} = \underline{\underline{1.9759}}$$

Decision rule: At $\alpha = 5\%$, accept H_0 if $t < t_{\alpha, n-1}$



$t_{\alpha, n-1} = t_{0.05, 7} = 1.895$

Shortcut to calculate mean → is in last page.

Conclusion: At $\alpha = 5\%$, $t = 1.9759 \nless 1.895$

∴ Reject H_0

2. ans $x \rightarrow$ IQ of student ; $x_i = 85, 96, 105, 102, 82, 89, 90.$

$n = 7$; $\alpha = 5\%$; $\mu = 100$

$H_0: \mu = 100$ [Population mean is 100]

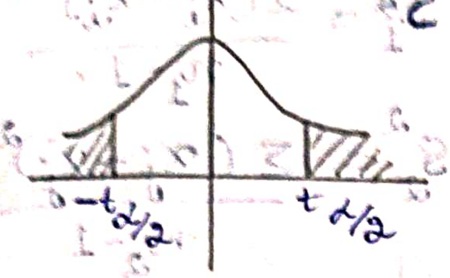
$H_1: \mu \neq 100$

Test statistics: $t = \frac{\bar{x}-\mu}{S/\sqrt{n}} = \frac{92.7142-100}{\frac{8.5967}{\sqrt{7}}} = -2.2423$

$\bar{x} = 92.7142$

$S^2 = 73.9047 \Rightarrow S = 8.5967$

Decision rule: At $\alpha = 5\%$ accept H_0 if $|t| < t_{\alpha/2, n-1}$



$t_{\alpha/2, n-1} = t_{0.025, 6} = 2.447$

Conclusion: At $\alpha = 5\%$, $|t| = 2.2423 < 2.447$

$\therefore$ Accept H_0.

1/1/25

3. ans $x \to$ Lifetime of a ~~som~~ fuse in days

$\bar{x} = 665$, $s = 65$.

$n = 17$

$\alpha = 5\%$

$\mu = 700$

S = S.D. for population

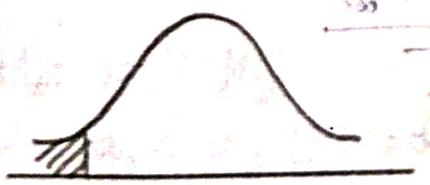
s = S.D. for sample

$H_0: \mu \geq 700$ [Company's claim is correct]

$H_1: \mu < 700$ [Claim is incorrect]

Test statistics: $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}} = \dfrac{665 - 700}{\frac{65}{\sqrt{16}}} = -2.1538$

Decision rule: At $\alpha = 5\%$ accept H_0 if $t > t_{\alpha, n-1}$



$t_{\alpha, n-1} = t_{0.05, 16} = 1.746$

Conclusion: At $\alpha = 5\%$, $t = -2.1538 \not> t_{\alpha, n-1} = -1.746$

$\therefore$ Reject H_0.

☆ TYPE-4: Difference between means (small sample)

Test statistics: $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$, $S^2 = \dfrac{\Sigma (x_i - \bar{x}_i)^2 + (x_j - \bar{x}_j)^2}{n_1 + n_2 - 2}$

or $S^2 = \dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$

$s_1^2 = \dfrac{\Sigma (x_i - \bar{x}_i)^2}{n_1}$, $s_2^2 = \dfrac{\Sigma (x_j - \bar{x}_j)^2}{n_2}$

$$S^2 = \frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}$$ if $$S_1^2 = \frac{\Sigma(x_i-\bar{x}_i)^2}{n_1-1}$$

$$S_2^2 = \frac{\Sigma(x_j-\bar{x}_j)^2}{n_2-1}$$

$H_0: \mu_1 = \mu_2$

H_1:

$\mu_1 \neq \mu_2$ — $|t| < t_{\alpha/2, n_1+n_2-2}$

$\mu_1 < \mu_2$ — $t > t_{\alpha, n_1+n_2-2}$

$\mu_1 > \mu_2$ — $t < t_{\alpha, n_1+n_2-2}$

4.ans $x_1, x_2 \rightarrow$ survival time of mice with treatment & without treatment respectively.

$n_1 = 5,\ n_2 = 4,\ \alpha = 5\%$

$H_0: \mu_1 \leq \mu_2$

$H_1: \mu_1 > \mu_2$ [Treatment is effective]

Test statistics: $$t = \frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

Calci.

$\sigma_x^2 = s^2$

$\sigma_x = s$

$\bar{x}_1 = 2.86$

$s_1^2 = 3.1064$

$\bar{x}_2 = 2.075$

$s_2^2 = 1.0218$

$$S^2 = \frac{5(3.1064)+4(1.0218)}{5+4-2}$$

$S^2 = 2.8027$

$S = 1.67412$

$$t = \frac{(2.86 - 2.075) - (0)}{1.6741\sqrt{\frac{1}{5}+\frac{1}{4}}} \Rightarrow t = 0.699$$

Decision Rule: At $\alpha = 5\%$, accept H_0 if $t < t_{\alpha, n_1+n_2-2}$

$t_{0.05, 7} = 1.895$

t_{α, n_1+n_2-2}

Conclusion: At $\alpha = 5\%$, $t = 0.699 < t_{\alpha, n_1+n_2-2} = 1.895$

$\therefore$ Accept H_0.

---

3. (in a B) A group of 5 patients treated with medicine "A" weigh 42, 39, 48, 60 & 41 kgs. A second group of 7 patients from the same hospital treated with medicine "B" weigh 38, 42, 56, 64, 68, 69 & 62 kgs. Do you agree with the claim that the medicine "B" increases the weight significantly?

ans: $x_1, x_2 \Rightarrow$ weight of patient treated with medicine A & B respectively.

$H_0: \mu_1 \geq \mu_2$

$H: \mu_1 < \mu_2$ [Medicine B increases weight]

Test statistics: $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

$\bar{x}_1 = 46$

$\bar{x}_2 = 57$

$s_1^2 = 58$

$s_2^2 = 132.2857$

$\cancel{S^2 = (57-46) - (0)}$

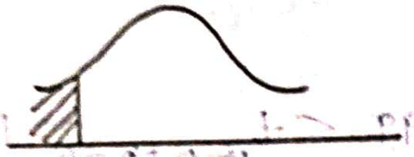
$$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$$

$$S^2 = \frac{5(58) + 7(132.2857)}{5+7-2} = 121.59$$

$S = 11.0272$

$$t = \frac{(46-57)}{11.0272\sqrt{\frac{1}{5}+\frac{1}{7}}} = -1.7036$$

Decision Rule: At $\alpha = 1\%$, accept H_0 if $t > t_{\alpha, n_1+n_2-2}$



t_{α, n_1+n_2-2}

$t_{0.01, 10} = -2.764$

Conclusion: At $\alpha = 1\%$ $t = -1.7036 > t_{0.01, 10} = -2.764$

$\therefore$ Accept H_0.

2.ans $x_1, x_2 \rightarrow$ Scores of children from group A & B respectively

$n_1 = n_2 = 8$, $\alpha = 5\%$

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$ [Difference between mean score of A & B is significant]

Test statistics: $$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

$\bar{x}_1 = 5.625$

$\bar{x}_2 = 5$

$s_1^2 = 1.7343$

$s_2^2 = 2$

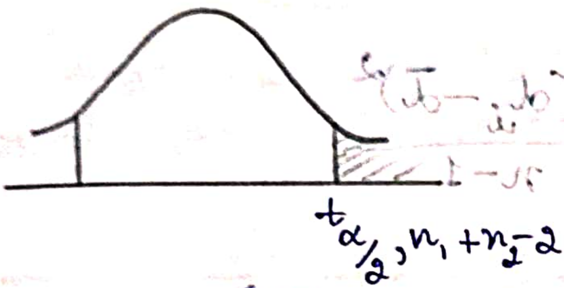
$$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2} = \frac{8(1.7343)+8(2)}{8+8-2}$$

$S^2 = 2.1338$

$S = 1.4607$

$$t = \frac{(5.625-5)-0}{1.4607\sqrt{\frac{1}{8}+\frac{1}{8}}} = 0.8557$$

Decision Rule: At $\alpha = 5\%$, accept H_0 if $|t| < t_{\alpha/2, n_1+n_2-2}$



$t_{\alpha/2, n_1+n_2-2} = t_{0.025,14} = 2.145$

Conclusion: At $\alpha = 5\%$, $|t| = 0.8557 < t_{\alpha/2, n_1+n_2-2} = 2.145$

$\therefore$ Accept H_0.

1.ans $x_1, x_2 \rightarrow$ Life of battery in days from A, B respectively

$\bar{x}_1 = 780$, $\bar{x}_2 = 750$, $n_1 = 9$, $n_2 = 8$, $s_1 = 50$, $s_2 = 45$ $s \rightarrow$ as SD of sample

$H_0: \mu_1 \leq \mu_2$

$H_1: \mu_1 > \mu_2$ [Battery from A are superior]

Test statistics: $t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$

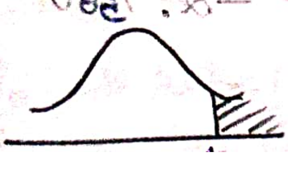
$$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2} = \frac{9(50)^2 + 8(45)^2}{9+8-2} = 2580$$

$$S = 50.7937$$

$$t = \frac{(780-750)}{50.7937\sqrt{\frac{1}{9}+\frac{1}{8}}} = 1.2155$$

Decision Rule: At $\alpha = 1\%$, accept H_0 if $t < t_{\alpha, n_1+n_2-2}$

$t_{0.01, 15} = 2.602$



Conclusion : At $\alpha = 1\%$, $t = 1.2155 < t_{\alpha, n_1+n_2-2} = 2.602$

$\therefore$ Accept H_0.

☆ TYPE-5: Paired t-test

Test statistics:

$$t = \frac{\bar{d} - \mu_d}{S_d/\sqrt{n}}, \quad S_d^2 = \frac{\sum (d_i - \bar{d})^2}{n-1}$$

or

$$t = \frac{\bar{d} - \mu_d}{S_d/\sqrt{n-1}}, \quad S_d^2 = \frac{\sum (d_i - \bar{d})^2}{n}$$

$H_0: \mu = \mu_d$

H_1:

$\mu \neq \mu_d$	$\mu < \mu_d$	$\mu > \mu_d$
$\lvert t \rvert < t_{\frac{\alpha}{2}, n-1}$	$t > t_{\alpha, n-1}$	$t < t_{\alpha, n-1}$

☆ Numericals:

1.ans $x_1, x_2 \rightarrow$ Marks of students before & after coaching session respectively.

$n = 10, \alpha = 5\%$

$H_0: \mu_1 \geq \mu_2$ $\quad \mu_1 < \mu_2 \Rightarrow \mu_1 - \mu_2 < 0 = \mu_d$

$H_1: \mu_1 < \mu_2$ [coaching is effective] ※ Take $\mu_d = 0$ in paired t-test

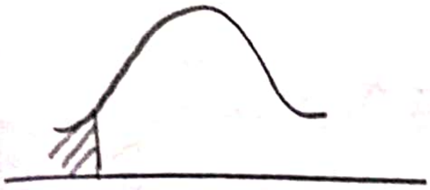
Test statistics: $d_i = -9, -10, +4, -10, -4, 3, -4, 0, -8, -8$

difference b/w before & after

$\bar{d} = -4.6$; $S_d = 5.3166$

$$t = \frac{\bar{d} - \mu_d}{S_d/\sqrt{n}} = \frac{-4.6}{\frac{5.3166}{\sqrt{10}}} = -2.7360$$

Decision rule: At $\alpha = 5\%$, accept H_0 if $t \geq t_{\alpha, n-1}$



$$t_{0.05, 9} = -1.833$$

$t_{0.05,9}$

Conclusion: At $\alpha = 5\%$, $t = -2.736 \ngtr t_{\alpha, n-1} = -1.833$

$\therefore$ Reject H_0.

2.ans $x_1, x_2 \rightarrow$ Scores of soldiers on day 1 & day 2 respectively

$n = 8$, $\alpha = 5\%$

$H_0: \mu_1 \geq \mu_2$

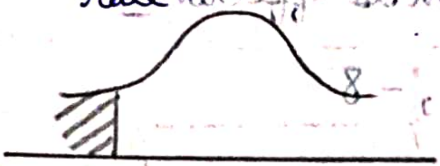
$H_1: \mu_1 < \mu_2$ [Performance has improved on second day]

Test statistics: d_i = 3, −6, −5, −4, 6, 3, −5, 0

$$\bar{d} = -1.75, \quad S_d = 4.3342$$

$$t = \frac{\bar{d} - \mu_d}{S_d/\sqrt{n}} = \frac{-1.75 - 0}{4.3342/\sqrt{8}} = \underline{\underline{-1.1420}}$$

Decision rule: At $\alpha = 5\%$, accept H_0 if $t \geq t_{\alpha, n-1}$



$$t_{\alpha, n-1} = t_{0.05, 7} = -1.895$$

$t_{\alpha, n-1}$

Conclusion: At $\alpha = 5\%$, $t = -1.1420 > t_{\alpha, n-1} = -1.895$

$\therefore$ Accept H_0.

2/1/25

4.ans $x_1, x_2 \rightarrow$ B.P. of a woman before & after the intake of drug.

$n = 5$, $\alpha = 1\%$

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$ [Significant change in BP]

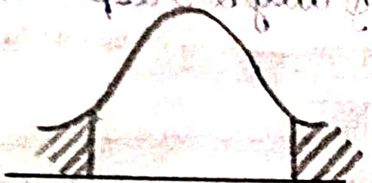
Test statistics: d_i: $-10, 2, 0, -4, 4$

$\bar{d} = -1.6$

$S_d = 5.5497$

$$t = \frac{\bar{d} - \mu_d}{S_d/\sqrt{n}} = \frac{-1.6 - 0}{5.5497/\sqrt{5}} = -0.6446$$

Decision Rule: At $\alpha = 1\%$, accept H_0 if $|t| < t_{\frac{\alpha}{2}, n-1}$



$t_{\alpha/2, n-1} = t_{0.005, 4} = 4.604$

Conclusion: At $\alpha = 1\%$, $|t| = 0.6446 < t_{0.005,4} = 4.604$

$\therefore$ Accept H_0.

3.ans $x_1, x_2 \Rightarrow$ No. of deals/month closed (by an employee) before & after attending training program.

$n = 5$, $\alpha = 5\%$

$H_0 : \mu_1 \geq \mu_2$ [Training program is not effective]

$H_1 : \mu_1 < \mu_2$ [Training program is effective]

Test statistics: d_i: $-3, 7, -1, 2, -8$

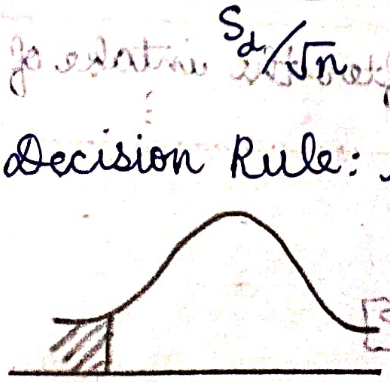
$\bar{d} = -0.6$

$S_d = 5.5946$

$$t = \frac{\bar{d} - \mu_d}{S_d/\sqrt{n}} = \frac{-0.6 - 0}{5.5946/\sqrt{5}} = -0.2398$$

Decision Rule: At $\alpha = 5\%$, accept H_0 if $t > t_{\alpha, n-1}$

$t_{\alpha, n-1} = t_{0.05, 4} = -2.132$



$t_{\alpha, n-1}$

Conclusion: At $\alpha = 5\%$, $t = -0.2398 > t_{\alpha, n-1} = -2.132$.

$\therefore$ Accept H_0.

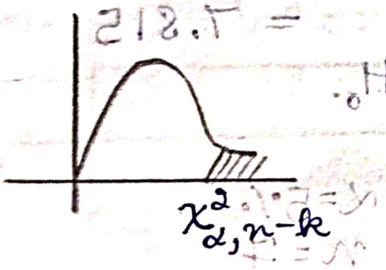
☆ Chi-Square ~~Distribution~~ Test: (TYPE-6)

→ Chi-Square distribution (goodness of fit): This is a non-parametric test where we test for goodness of fit or homogeneity or Independence of attributes.

→ Goodness of fit : H_0: Distribution is a good fit
H_1: Distribution is not a good fit

Test statistics:
$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

Decision rule: At $\alpha\%$ accept H_0 if $\chi^2 < \chi^2_{\alpha, n-k}$



☆ Problems

1. Four coins were tossed 200 times & the following results are obtained.

Check whether Poisson distribution is a good fit for the given data at $\alpha = 5\%$.

ans H_0: Poisson dist. is a good fit
H_1: —11— is not a good fit

Test statistic:

n	f (O_i)	$P(x)$	Expected (E_i)	$(O_i - E_i)^2/E_i$
0	122	0.6065	121.3 ~ 121	0.0083
1	60	0.3032	60.64 ~ 61	0.0164
2	15	0.0758	15.16 ~ 15	0
3	2	0.0126	2.52 ~ 2	0
4	1	0.0015	0.3 ~ 0.1	0

$E_i \to$ cannot be 0, or else, we get $\frac{\square}{0}$ error

$$\lambda = \frac{\Sigma f x}{\Sigma f} = \frac{100}{200} = 0.5$$

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \frac{e^{-0.5}(0.5)^x}{x!}$$

$$\chi^2 = \Sigma\left[(O_i - E_i)^2 / E_i\right] = 0.0247$$

for Poisson Distribution $\lambda = \frac{\Sigma f x}{\Sigma f}$, Σf = Total (adjacent sum)

NOTE: $k \Rightarrow$ no. of assumptions made (here always 2).

Decision rule: At $\alpha = 5\%$ accept H_0 if $\chi^2 < \chi^2_{\alpha, n-k}$

$$\chi^2_{\alpha, n-k} = \chi^2_{0.05, 5-2} = \chi^2_{0.05, 3}$$
$$= 7.815$$

$\therefore$ Accept H_0.

---

2. ans H_0: Poisson dist. is good fit $\qquad \alpha = 5\%$

H_1: —11— is not a good fit $\qquad n = 7$

Test statistics:

n	f (O_i)	$P(x)$	Expected (E_i)	$(O_i - E_i)^2/E_i$
0	5	0.0907	7.256 $\simeq$ 7	0.5714
1	18	0.2177	17.416 $\sim$ 18	0
2	28	0.2612	20.896 $\sim$ 21	2.3333
3	12	0.2090	16.72 $\sim$ 17	1.4706
4	7	0.1254	10.032 $\sim$ 10	0.9
5	6	0.0602	4.816 $\sim$ 5	0.2
6	4	0.0241	1.928 $\sim$ 2	2

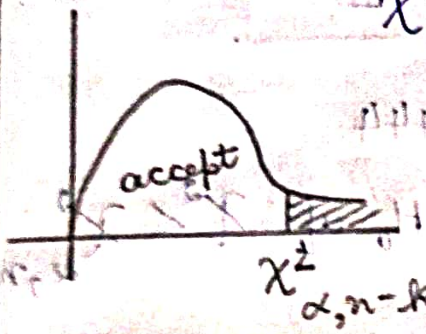
$$\lambda = \frac{\Sigma f x}{\Sigma f} = \frac{192}{80} = 2.4$$

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \frac{e^{-2.4}(2.4)^x}{x!}$$

$$\chi^2 = \Sigma\,[\,(O_i - E_i)^2/E_i\,] = 7.4753$$

Decision rule: At $\alpha = 5\%$, accept H_0 if $\chi^2 < \chi^2_{\alpha, n-k}$



$$\chi^2_{\alpha, n-k} = \chi^2_{0.05, 7-2} = \chi^2_{0.05, 5} = 11.070$$

$\therefore\ \chi^2 < \chi^2_{\alpha, n-k}$

$\therefore$ Accept H_0.

3/1/25

3.ans H_0: ~~Exam~~ination result is in stated proportion.

H_1: —II— is not in stated proportion

Here we assumed the ratio so, $k = 1$

Test statistic:

		O_i	Expected	$(O_i - E_i)^2/E_i$
Failed	$\cancel{\frac{220}{500}}$	220	$\frac{4}{10} \times 500 = 200$	2
Third	$\cancel{\frac{170}{500}}$	170	$\frac{3}{10} \times 500 = 150$	2.6667
Second	$\cancel{\frac{90}{500}}$	90	$\frac{2}{10} \times 500 = 100$	1
First	$\cancel{\frac{20}{500}}$	20	$\frac{1}{10} \times 500 = 50$	18

$$\chi^2 = \Sigma\,[\,(O_i - E_i)^2/E_i\,] = 23.6667$$

Decision Rule: At $\alpha = 1\%$ accept H_0 if $\chi^2 < \chi^2_{\alpha, n-k}$

$$\chi^2_{\alpha, n-k} = \chi^2_{0.01, 3} = 11.345$$

Conclusion: At $\alpha = 1\%$ as $\chi^2 = 23.6667 \nless \chi^2_{0.01,3} = 11.345$

$\therefore$ Reject H_0.

4.ans H_0: The data is in stated proportion

H_1: —II— is not in stated proportion

Test statistic:

	O_i	E_i	$(O_i-E_i)^2/E_i$
Red	34	$\frac{9}{16}\times 64 = 36$	0.1111
Black	10	$\frac{3}{16}\times 64 = 12$	0.3333
White	20	$\frac{4}{16}\times 64 = 16$	1

$$\chi^2 = \Sigma\left[(O_i-E_i)^2/E_i\right] = 1.4444$$

Decision Rule: At $\alpha = 5\%$, accept H_0 if $\chi^2 < \chi^2_{\alpha, n-k}$

$$\chi^2_{0.05, 2} = 5.991$$

Conclusion: At $\alpha = 5\%$ as $\chi^2 = 1.4444 < \chi^2_{0.05,2} = 5.991$

$\therefore$ Accept H_0

★ TYPE 7: F-Test KEYWORD: variance

(Test for equality of variance)

$H_0: \sigma_1^2 = \sigma_2^2$ [equal variance]

$H_1: \sigma_1^2 \neq \sigma_2^2$ [unequal variance]

Test statistic:

$$F = \frac{S_1^2}{S_2^2} \quad [\text{if } S_1^2 > S_2^2]$$

or

$$F = \frac{S_2^2}{S_1^2} \quad [\text{if } S_2^2 > S_1^2]$$

$$S_1^2 = \frac{\Sigma(x_i - \bar{x}_i)^2}{n_1 - 1} \quad \text{or} \quad S^2 = \frac{ns^2}{n-1}$$

Decision Rule: At $\alpha\%$ accept H_0 if

$$F < F_{\alpha, n_1-1, n_2-1}$$

1. ans $x_1, x_2 \rightarrow$ Time taken by worker by Method 1, Method 2 respectively.

$n_1 = 6, n_2 = 7, \alpha = 5\%$

$H_0 : \sigma_1^2 = \sigma_2^2$ [variance does not differ significantly]

$H_1 : \sigma_1^2 \neq \sigma_2^2$ [variance differs significantly]

Test statistic:

$$S_1^2 = \frac{\Sigma (x_i - \bar{x}_i)^2}{n_1 - 1} \qquad S_2^2 = \frac{\Sigma (x_i - \bar{x}_i)^2}{n_2 - 1}$$

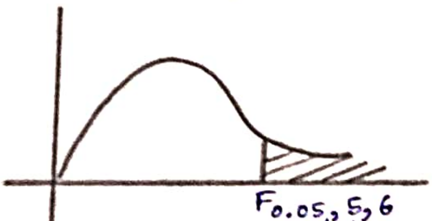
$$\bar{x}_1 = 22.3333 \qquad \bar{x}_2 = 34.4285$$

$$S_1^2 = 16.2667 \qquad S_2^2 = 22.2857$$

$$S_2^2 > S_1^2$$

$$F = \frac{S_2^2}{S_1^2} = \frac{22.2857}{16.2667} = 1.3700$$

Decision rule: At $\alpha = 5\%$, accept H_0 if $F < F_{\alpha, n_1-1, n_2-1}$



$$F_{0.05,5,6} = 4.39$$

Conclusion: At $\alpha = 5\%$. as $F = 1.37 < F_{0.05,5,6} = 4.39$

$\therefore$ Accept H_0.

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1. ans $x_1, x_2 \rightarrow$ No. of Post graduates passed out from a state & private university respectively.

$n_1 = 5, n_2 = 6, \alpha = 5\%$

$H_0 : \sigma_1^2 = \sigma_2^2$ [Variance is equal]

$H_1 : \sigma_1^2 \neq \sigma_2^2$ [Variance is not equal]

Test statistic:

$$S_1^2 = \frac{\Sigma (x_i - \bar{x}_i)^2}{n_1 - 1}$$

$$\bar{x}_1 = 8230 \qquad \bar{x}_2 = 7940$$

$$S_1^2 = 15750 \qquad S_2^2 = 10920$$

$$S_1^2 > S_2^2$$

$$F = \frac{S_1^2}{S_2^2} = 1.4423$$

Decision rule: At $\alpha = 5\%$ accept H_0 if $F < F_{\alpha, n_1-1, n_2-1}$

$$F_{0.05, 4, 5} = 5.19$$

Conclusion: At $\alpha = 5\%$ as $F = 1.4423 < F_{\alpha, n_1-1, n_2-1} = 5.19$,

$\therefore$ Accept H_0.