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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

December 2023 Supplementary Examinations

Programme: B.E. **Semester: III**

Branch: Common to all branches except CS, IS and AI&ML. **Duration: 3 hrs.**

Course Code: 22MA3BSTFN **Max Marks: 100**

Course: Transform Calculus, Fourier series and Numerical Techniques

Instruction: Answer any FIVE full questions, choosing one full question from each unit.

UNIT - I

1 a) Find $L\left[\frac{\cos at - \cos bt}{t} + t\sin at\right]$ **6**

b) If a periodic function of period $2a$ is defined as **7**

$$f(t) = \begin{cases} t, & if\ \ 0 \le t \le a \\ 2a - t, & if\ \ a \le t \le\le 2a \end{cases}$$

then show that $L[f(t)] = \frac{1}{s^2}\tanh\left(\frac{as}{2}\right)$.

c) In a single loop LR circuit the current I builds up at the rate given by $L\frac{dI}{dt} + RI = E(t)$. Determine the current I(t), when $L = 1\text{H}, R = 1\Omega$ and $E(t) = f(x) = \begin{cases} t, & 0 < t < 1 \\ 0, & t > 1 \end{cases}$ subject to the initial condition $I(0) = 0$. **7**

OR

2 a) Find the inverse Laplace transform of the function $\text{F}(s) = \frac{6s+3}{s^4+5s^2+4}$. **6**

b) Express the function **7**

$$f(t) = \begin{cases} \cos t, & if\ \ 0 < t < \pi \\ 1, & if\ \pi < t < 2\pi \\ \sin t, & if\ t > 2\pi \end{cases}$$

in terms of Unit step function and find its Laplace transform.

c) Solve $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 12t^2e^{-3t}, y(0) = 0 = y'(0)$ by using Laplace transform technique. **7**

UNIT - II

3 a) Obtain the complex form of Fourier series of the periodic function $f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases}$. **6**

b) Obtain the Fourier series of $f(x) = \frac{\pi - x}{2}$ in $(0, 2\pi)$ and hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$. **7**

c) Obtain the Fourier series of y up to the terms containing first harmonic from the following table. **7**

x	0	1	2	3	4	5	6
y	9	18	24	28	26	20	9

UNIT - III

4 a) Obtain the Fourier sine transform of the function **6**

$$f(x) = \begin{cases} 4x, & if\ 0 < x < 1 \\ 4 - x, & if\ 1 < x < 4 \\ 0, & if\ x > 4 \end{cases}$$

b) Find the complex Fourier transform of $f(x) = e^{-a^2x^2}, a > 0$. Hence deduce that $e^{-x^2/2}$ is self-reciprocal in respect of the complex Fourier transform. **7**

c) By employing the convolution theorem, show that the inverse Fourier transform of $e^{-s^2/2}$ is $\frac{1}{\sqrt{2\pi}}e^{-x^2/2}$. **7**

UNIT - IV

5 a) Derive Crank-Nicolson formula for the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$. **6**

b) Solve $u_{tt} = 4u_{xx}$ with boundary conditions $u = 0\ at\ x = 0,\ t > 0$ and $u = 0\ at\ x = 4,\ t > 0$ and initial conditions $u = x(4 - x)$ and $\frac{\partial u}{\partial t} = 0$ at $t = 0,\ 0 \le x \le 4$ taking $h = 1$ and $k = \frac{1}{2}$ up to $t = 2$. **7**

c) Solve by Bendre-Schmidt method $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, 0 \le x \le 1$ subjected to the conditions $u(0,t) = 0,\ u(1,t) = 0$ and $u(x,0) = 100x(1-x)$ taking $h = 0.25$ for three time steps. **7**

UNIT - V

6 a) Derive an Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **6**

b) Find the Z – transform of the following: **7**
(i) $3n - 4\sin\frac{n\pi}{4} + 5a$, (ii) $\cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$.

c) Find the inverse Z – transform of $\frac{3z^2-18z+26}{(z-2)(z-3)(z-4)}$. **7**

OR

7 a) Apply Z-transforms to solve the difference equation $u_{n+2} - 5u_{n+1} + 6u_n = 1$ with $u_0 = 0,\ u_1 = 1$. **6**

b) Find the extremal of the functional $I = \int_0^{\pi/2}(y'^2 - y^2 + 4y\cos x)\,dx$ under the end conditions $y(0) = 0;\ y\left(\frac{\pi}{2}\right) = 0$. **7**

c) A heavy cable hangs freely under gravity between two fixed points. Show that the shape of the cable is a Catenary. **7**

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