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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

April 2023 Semester End Main Examinations

Programme: B.E. **Semester: III**

Branch: Common to all branches except CS, IS and AI&ML. **Duration: 3 hrs.**

Course Code: 22MA3BSTFN **Max Marks: 100**

Course: Transform Calculus, Fourier series and Numerical Techniques **Date: 10.04.2023**

Instruction: Answer any FIVE full questions, choosing one full question from each unit.

UNIT - I

1 a) Find the Laplace transform of the following functions (i) $f(t) = te^{2t}\sin(3t)$ (ii) $f(t) = \frac{\sin(3t)}{t}$ **6**

b) Find the Laplace transform of square wave function $f(t)$ of period a. where $f(t) = \begin{cases} 1 & if \quad 0 < t < \frac{a}{2} \\ -1 & if \quad \frac{a}{2} < t < a \end{cases}$. **7**

c) Solve $y''' - 3y'' + 3y' - y = t^2 e^t$ given $y(0) = 1,\ y'(0) = 0,\ y''(0) = 1$ by the method of Laplace transform. **7**

OR

2 a) Find the inverse Laplace transform of the following functions (i) $f(s) = \frac{2s-3}{s^2+4s+13}$ (ii) $f(s) = \frac{(s+1)^2}{(s+2)^4}$ **6**

b) Express the function $f(t) = \begin{cases} t^2, & 0 < t < 2 \\ 4t, & 2 < t < 4 \\ 8, & t > 4 \end{cases}$ in terms of unit step function and hence find its Laplace transform. **7**

c) In an electrical circuit with e.m.f E(t), inductance L_1, resistance R and capacitance C the current i builds up at the rate given by $L_1 \frac{di}{dt} + Ri + \frac{1}{C}\int_0^t i(t)\,dt = E(t)$. Determine the current i(t) when $L_1 = 0.1H,\ R = 3\Omega,\ C = 0.05F$ and $E(t) = 100\left[u(t-1) - u(t-2)\right]$ with the zero initial condition. **7**

UNIT - II

3 a) Obtain the complex form of Fourier series for the periodic function $f(x) = \begin{cases} 1+2x & \text{in} \quad -3 < x < 0 \\ 1-2x & \text{in} \quad 0 < x < 3 \end{cases}$ **6**

b) Find the Fourier series of the periodic function $f(x)=\begin{cases}1+\frac{4x}{3} & \text{in } \frac{-3}{2}<x<0\\ 1-\frac{4x}{3} & \text{in } 0<x<\frac{3}{2}\end{cases}$ and hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots\ldots=\frac{\pi^2}{8}$ **7**

c) The displacement y of a part of a mechanism is tabulated with corresponding angular movement θ of the crank. Express y as a Fourier series up to the first harmonic. **7**

θ	0^0	60^0	120^0	180^0	240^0	300^0	360^0
y	7.9	7.2	3.6	0.5	0.9	6.8	7.9

UNIT - III

4 a) Find the Fourier transform of $f(x)=\begin{cases}1-|x| & \text{when } |x|\le 1\\ 0 & \text{when } |x|>1\end{cases}$ **6**

b) Find the inverse Fourier sine transform of $f_s(s)=\frac{e^{-as}}{s},\ a>0$ **7**

c) Apply the convolution theorem to find the inverse Fourier transform of $e^{-s^2/2}$ if the Fourier transform of $e^{-a^2x^2}$ is $\frac{\sqrt{\pi}}{a}e^{-s^2/4a^2}$. **7**

UNIT - IV

5 a) Derive Crank-Nicolson formula for the solution of one-dimensional heat equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$. **6**

b) Solve the initial boundary value problem $u_t=4u_{xx}$ subject to the conditions $u(0,t)=0=u(8,t), t>0$ and $u(x,0)=4x-\frac{x^2}{2}, 0\le x\le 8.$ Carryout the computations up to one-time level by taking $h=1$ and $k=0.1$. **7**

c) Apply explicit three level formula to solve the wave equation $\frac{\partial^2 u}{\partial t^2}=4\frac{\partial^2 u}{\partial x^2}$ subject to the boundary conditions $u(0,t)=u(1,t)=0,\ t\ge 0,$ initial conditions $u(x,0)=\begin{cases}\frac{5x}{3} & \text{for } 0<x\le\frac{3}{5}\\ \frac{5(1-x)}{2} & \text{for } \frac{3}{5}<x<1\end{cases}$ and $u_t(x,0)=0$ up to two time levels taking $h=\frac{1}{5}$ and $k=\frac{1}{10}$. **7**

UNIT - V

6 a) Derive an Euler's equation in the form $\frac{\partial f}{\partial y}-\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)=0.$ **6**

b) Find the extremal of the functional $\int_0^{\frac{\pi}{2}}\left(y'^2-y^2+4y\cos x\right)dx$ with $y(0)=y(\frac{\pi}{2})=0$. **7**

c) Find the Z-transform of $\cos(n\theta)$ and $\sin(n\theta)$. Hence evaluate $Z_T\left(4\sin\frac{n\pi}{4}\right)$. **7**

OR

7 a) Find the inverse Z-transform of $\frac{2z^2+3z}{(z+2)(z-4)}$ **6**

b) Show that the shape of the heavy cable which hangs freely under gravity between two fixed points is a catenary. **7**

c) Apply Z-transform to solve the difference equation $u_{n+2}+2u_{n+1}+u_n=n$ given $u_0=0, u_1=0$. **7**

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