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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

September / October 2024 Supplementary Examinations

Programme: B.E. | **Semester: III**
Branch: All Branches (Except CSE stream) | **Duration: 3 hrs.**
Course Code: 22MA3BSTFN | **Max Marks: 100**
Course: Transform Calculus, Fourier series and Numerical Techniques

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - I	**CO**	**PO**	**Marks**
1	a)	Find the Laplace transform of $2^t + \frac{\cos 2t - \cos 3t}{t} + t\sin t$.	CO1	PO1	**6**
	b)	Find the Laplace transform of the full wave rectifier $f(t) = E\sin\omega t$, $0 < t < \pi/\omega$ having period π/ω.	CO2	PO1	**7**
	c)	Solve the initial value problem $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-t}$, $y(0) = y'(0) = 0$ by using Laplace transform.	CO1	PO1	**7**
		OR			
2	a)	Find the Laplace transform of $te^{2t} - \frac{2\sin 3t}{t}$.	CO1	PO1	**6**
	b)	Express the function $f(t) = \begin{cases} 1, & 0 < t \le 1 \\ t, & 1 < t \le 2 \\ t^2, & t > 2 \end{cases}$ in terms of unit step function and hence find its Laplace transform.	CO1	PO1	**7**
	c)	Find the inverse Laplace transform of the function $\frac{(s+2)e^{-s}}{(s+1)^4}$.	CO1	PO1	**7**
		UNIT - II			
3	a)	Find the Fourier series of $f(x) = e^{-x}$ over the interval $(0, 2\pi)$.	CO1	PO1	**6**

		b)	Obtain the Fourier series for the function $f(x)=\begin{cases}\pi x & 0\le x\le 1\\ \pi(2-x) & 1\le x\le 2\end{cases}$ and hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+...=\frac{\pi^2}{8}$.	CO1	PO1	**7**
		c)	Obtain the Fourier series expansion up to first harmonic for the given data: x: 0, 1, 2, 3, 4, 5 y: 4, 8, 15, 7, 6, 2	CO1	PO1	**7**
			UNIT - III			
4		a)	Find the Fourier sine transform of $\frac{e^{-ax}}{x}$.	CO1	PO1	**6**
		b)	Solve the integral equation $\int_0^\infty f(\theta)\cos a\theta\, d\theta=\begin{cases}1-a & 0\le a\le 1\\ 0 & a>1\end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin^2 t}{t^2}dt$.	CO1	PO1	**7**
		c)	Apply convolution theorem to show that $F^{-1}\left(e^{-s^2/2}\right)=\frac{1}{\sqrt{2\pi}}e^{-x^2/2}$.	CO1	PO1	**7**
			UNIT - IV			
5		a)	Derive Schmidt explicit formula for the solution of one-dimensional heat equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$.	CO1	PO1	**6**
		b)	Solve $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(x,0)=\sin\pi x$, $0\le x\le 1$ $u(0,t)=0=u(1,t)$. Carry out the computations for two-time levels taking $h=1/3$ and $k=1/36$.	CO1	PO1	**7**
		c)	The transverse displacement u of a point at a distance x from one end at any time t of a vibrating string satisfies the equation $u_{tt}=4u_{xx}$ subject to the conditions $u(0,t)=0$, $u(4,t)=0, t\ge 0$, $u(x,0)=x(4-x)$ $0\le x\le 4$ and $u_t(x,0)=0$. Solve this equation numerically up to two-time levels, with $h=1$ and $k=0.5$.	CO2	PO1	**7**
			UNIT - V			
6		a)	Find the extremal of the functional $\int_{x_1}^{x_2}\left(y'^2-y^2+2y\sec x\right)dx$.	CO1	PO1	**6**

		b)	Find the Z-transform of $2n+\sin\left(\frac{n\pi}{4}\right)+1$.	CO1	PO1	**7**
		c)	Solve the difference equation $y_{n+2}-4y_{n+1}+3y_n=1$ with $y_0=0$, $y_1=1$.	CO1	PO1	**7**
			OR			
	7	a)	A heavy cable hangs freely under gravity between two fixed points. Show that the shape of the cable is a catenary.	CO2	PO1	**6**
		b)	Show that the equation of the curve joining the points $(1,0)$ and $(2,1)$ for which $I=\int_1^2 \frac{\sqrt{1+y'^2}}{x}\,dx$ is an extremum is a circle.	CO2	PO1	**7**
		c)	Find the inverse Z-transform of $\frac{2z^2+3z}{(z+2)(z-4)}$.	CO1	PO1	**7**

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