U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

August 2023 Semester End Make-Up Examinations

Programme: B.E. — **Semester: III**

Branch: Common to all branches except CS, IS and AI&ML. — **Duration: 3 hrs.**

Course Code: 22MA3BSTFN — **Max Marks: 100**

Course: Transform Calculus, Fourier series and Numerical Techniques — **Date: 16.08.2023**

Instruction: Answer any FIVE full questions, choosing one full question from each unit.

UNIT - I

1 a) Find $L\left[2^t + \dfrac{\cos 2t - \cos 3t}{t} + t\sin t\right]$ **6**

b) Find the inverse Laplace transform of $\dfrac{4s+5}{(s+1)^2(s+2)}$ **7**

c) Express the function $f(t)=\begin{cases}\cos t, & 0<t<\pi\\ \cos 2t, & \pi<t<2\pi\\ \cos 3t, & t>2\pi\end{cases}$ in terms of unit step function and hence find its Laplace transform. **7**

OR

2 a) Find the inverse Laplace transforms of (i) $\dfrac{2s-1}{s^2+4s+29}$ (ii) $\dfrac{e^{-4s}}{s^2(s+3)}$ **6**

b) If $f(t)=\begin{cases}a, & 0\le t\le a\\ -a, & a<t\le 2a\end{cases}$ and $f(t+2a)=f(t)$, show that $L[f(t)]=\dfrac{a}{s}\tanh\left(\dfrac{as}{2}\right)$. **7**

c) Apply Laplace transform to solve the differential equation $\dfrac{d^2y}{dt^2}+5\dfrac{dy}{dt}+6y=5e^{2t}$ subjected to the conditions $y(0)=2,\quad y'(0)=1$. **7**

UNIT - II

3 a) Obtain the Fourier series of the periodic function $f(x)=x$ in $(-\pi,\pi)$ and hence deduce that $\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+-\ldots\ldots$ **6**

b) Obtain the complex form of Fourier series of the periodic function $f(x)=e^{-x}$ in $(-1,1)$. **7**

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

c) Obtain the Fourier series neglecting the terms higher than first harmonics for the given data. 7

x°	0	45	90	135	180	225	270	315
y	2	1.5	1	0.5	0	0.5	1	1.5

UNIT - III

4 a) Find the Fourier transform of $f(x)=\begin{cases} a^2-x^2, & |x|\le a \\ 0, & |x|>a \end{cases}$. 6

b) Find the inverse Fourier cosine transform of the function 7

$$f_c(s)=\begin{Bmatrix} 4s & 0<s<1 \\ 4-s & 1<s<4 \\ 0 & s>4 \end{Bmatrix}$$

c) Apply convolution theorem to find the inverse Fourier transform of $\frac{i}{(1+s^2)^2}$ given that $\frac{2}{1+s^2}$ is the Fourier transform of $e^{-|x|}$. 7

UNIT - IV

5 a) Derive Crank-Nicolson formula for the solution of one-dimensional heat equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$. 6

b) Solve the equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(x,0)=\sin \pi x$, $0\le x\le 1$, $u(0,t)=u(1,t)=0$ using Schmidt method taking $\boldsymbol{h=0.2}$ and $\boldsymbol{\alpha=\frac{1}{2}}$. 7

c) Find the solution of initial boundary value problem $\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$, $0\le x\le 1$, subject to the initial conditions $u(x,0)=\sin \pi x$, $u_t(x,0)=0$ $0\le x\le 1$ and the boundary conditions $u(0,t)=0$, $u(1,t)=0$, $t>0$ by taking $h=0.2$ and $k=0.2$. 7

UNIT - V

6 a) Derive Euler's equation in the form $\frac{\partial f}{\partial y}-\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)=0$. 6

b) Find the extremal of the functional $\int_0^{\frac{\pi}{2}}\left(y'^2-y^2+4y\cos x\right)dx$ with $y(0)=y(\frac{\pi}{2})=0$. 7

c) Solve $u_{n+2}+6u_{n+1}+9u_n=2^n$ with $u_0=0,\ u_1=0$ using Z- transforms. 7

OR

7 a) Find the Z- transform of i) $\cosh n\theta$ ii) $k^2 n^3$. 6

b) Find the inverse Z-transform of $\frac{3z^2+2z}{(5z-1)(5z+2)}$. 7

c) A uniform chain of fixed length l is freely suspended from two points so as to hang at rest under the action of gravity. Determine the shape of the curve. 7

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