U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

April 2024 Semester End Main Examinations

Programme: B.E. **Semester: III**
Course Code & Branch: **Duration: 3 hrs.**
23MA3BSTFN (Common to all Branches except Civil Engg. & CS-Stream) / **Max Marks: 100**
22MA3BSTFN (Common to all Branches except CS-Stream)
Course: Transform Calculus, Fourier Series and Numerical Techniques

Instructions:1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	*CO*	*PO*	Marks
1	a)	Find the Laplace transform of $f(t)=te^{-3t}\sin^3 t$.	*CO1*	*PO1*	**06**
	b)	Find the Laplace transform of the triangular wave function $f(t)$ of period $2a$ where $f(t)=\begin{cases} t & 0\le t\le a \\ 2a-t & a\le t\le 2a \end{cases}$ and hence deduce that $L\left[f(t)\right]=\frac{1}{s^2}\tanh\left(\frac{as}{2}\right)$.	*CO1*	*PO1*	**07**
	c)	Find the inverse Laplace transform of $F(s)=\frac{2s-1}{(s+1)(s^2-3s+2)}$.	*CO1*	*PO1*	**07**
		OR			
2	a)	Find the inverse Laplace transform of i) $F(s)=\frac{e^{-3s}}{(s-4)^2}$ ii) $F(s)=\frac{2s+5}{s^2+6s+34}$.	*CO1*	*PO1*	**06**
	b)	Express the function $f(t)=\begin{cases} \cos t & 0<t<\pi \\ \cos 2t & \pi\le t\le 2\pi \\ \cos 3t & t>2\pi \end{cases}$ in terms of Heaviside function and hence find its Laplace transform.	*CO1*	*PO1*	**07**
	c)	Solve the differential equation $\frac{d^2x}{dt^2}-2\frac{dx}{dt}+x=e^{2t}$ with $x(0)=0$ and $x'(0)=-1$ by the method of the Laplace transform.	*CO1*	*PO1*	**07**
		UNIT - 2			
3	a)	Obtain the complex form of the Fourier series for the function $f(x)=\begin{cases} -k & \text{for } -\pi<x<0 \\ k & \text{for } 0<x<\pi \end{cases}$	*CO1*	*PO1*	**06**
	b)	Obtain the Fourier series expansion for the periodic function $f(x)=x(2-x)$ in the interval $(0,2)$.	*CO1*	*PO1*	**07**

	c)	Compute the constant term and the amplitude of the first harmonic in the Fourier series expansion of $f(x)$ from the following table. x: 0, $\frac{\pi}{3}$, $\frac{2\pi}{3}$, π, $\frac{4\pi}{3}$, $\frac{5\pi}{3}$, 2π $f(x)$: 1.0, 1.4, 1.9, 1.7, 1.5, 1.2, 1.0	*CO1*	*PO1*	**07**
		UNIT - 3			
4	a)	Find the Fourier transform of $f(x)=\begin{cases}1-\lvert x\rvert & \text{for } \lvert x\rvert \le 1\\ 0 & \text{for } \lvert x\rvert > 1\end{cases}$.	*CO1*	*PO1*	**06**
	b)	Apply convolution theorem to find the inverse Fourier transform of $\frac{1}{(1+\alpha^2)^2}$, given $F\left[e^{-\lvert x\rvert}\right]=\sqrt{\frac{2}{\pi}}\left[\frac{1}{1+\alpha^2}\right]$.	*CO1*	*PO1*	**07**
	c)	Solve the integral equation $\int_0^\infty f(x)\cos(sx)\,dx=\begin{cases}1-s & \text{for } 0\le s\le 1\\ 0 & \text{for } s>1\end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin^2(t)}{t^2}\,dt$.	*CO1*	*PO1*	**07**
		UNIT - 4			
5	a)	Derive Schmidt explicit formula for the solution of one-dimensional heat equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$.	*CO1*	*PO1*	**06**
	b)	Solve the heat equation $2\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}$ subject to the conditions $u(0,t)=0=u(4,t),\ t\ge 0$ and $u(x,0)=x(4-x),\ 0<x<4$ using Bendre-Schmidt method by taking $h=1$ up to t =1.	*CO1*	*PO1*	**07**
	c)	Evaluate the pivotal values of the equation $\frac{\partial^2 u}{\partial t^2}=16\frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(0,t)=0=u(5,t),\ t\ge 0$ and $u(x,0)=x^2(5-x),\ \frac{\partial u}{\partial t}(x,0)=0$ taking $h=1,\ k=\frac{1}{4}$ up to two time-levels.	*CO1*	*PO1*	**07**
		UNIT - 5			
6	a)	Derive Euler's equation in the form $\frac{\partial f}{\partial y}-\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)=0$.	*CO1*	*PO1*	**06**
	b)	Find the curve on which the functional $\int_0^{\frac{\pi}{2}}\left(y'^2-y^2+2xy\right)dx$ with $y(0)=y\left(\frac{\pi}{2}\right)=0$ can be extremized.	*CO1*	*PO1*	**07**
	c)	Find the inverse Z-transform of $\frac{z}{(z+3)^2(z-2)}$.	*CO1*	*PO1*	**07**

			OR			
	7	a)	Find the *Z*-transform of $\cosh n\theta$ and hence find the *Z*-transform of $a^n \cosh n\theta$.	*CO1*	*PO1*	**06**
		b)	Prove that the shape of a heavy chain suspended under the gravity between two fixed points is a Catenary.	*CO1*	*PO1*	**07**
		c)	Solve the difference equation $6u_{n+2} - u_{n+1} - u_n = 0$, with $u_o = 0$ and $u_1 = 1$ using *Z*-transform.	*CO1*	*PO1*	**07**

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