U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

June / July 2024 Semester End Make-Up Examinations

Programme: B.E. — **Semester: III**
Course Code & Branch: — **Duration: 3 hrs.**
23MA3BSTFN (Common to all Branches except Civil Engg. & CS-Stream) / — **Max Marks: 100**
22MA3BSTFN (Common to all Branches except CS-Stream)
Course: Transform Calculus, Fourier Series and Numerical Techniques

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	CO	PO	Marks
1	a)	Find $L(t^2 e^{4t} \cosh 3t)$.	CO1	PO1	**06**
	b)	Prove that $L(f(t)) = \frac{E}{s}\tanh\left(\frac{as}{4}\right)$ where $f(t+a) = f(t)$, given $f(t) = \begin{cases} E & 0 \le t \le \frac{a}{2} \\ -E & \frac{a}{2} \le t \le a \end{cases}$.	CO1	PO1	**07**
	c)	A particle is moving with damping motion according to the law $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = 0$. If the initial position of the particle is at $y = 20$ and the initial speed is 10, find the displacement of the particle at any time 't' using Laplace transform.	CO1	PO1	**07**
		OR			
2	a)	Find the inverse Laplace transform of $F(s) = \frac{1}{s(s+1)(s+2)(s+3)}$.	CO1	PO1	**06**
	b)	Express the function $f(t) = \begin{cases} \cos(t) & 0 < t \le \pi \\ 1 & \pi < t \le 2\pi \\ \sin(t) & t > 2\pi \end{cases}$ in terms of the unit step function and hence find its Laplace transform.	CO1	PO1	**07**
	c)	Solve the differential equation $y''' + 2y'' - y' - 2y = 0$, with $y''(0) = y'(0) = 2, y(0) = 1$ by the Laplace transform method.	CO1	PO1	**07**
		UNIT - 2			
3	a)	Obtain the complex form of the Fourier series for the function $f(x) = \begin{cases} -k & \text{for } -\pi < x < 0 \\ k & \text{for } 0 < x < \pi \end{cases}$.	CO1	PO1	**06**
	b)	Obtain the Fourier series for the periodic function $f(x) = \frac{\pi - x}{2}$ in the interval $(0, 2\pi)$.	CO1	PO1	**07**

		c)	The following table gives the variations of periodic current over a period T. Show that there is a direct current part of 0.75amp in the variable current and obtain the amplitude of the first harmonic. <table><tr><td>t (sec)</td><td>0</td><td>T/6</td><td>T/3</td><td>T/2</td><td>2T/3</td><td>5T/6</td><td>T</td></tr><tr><td>A(amp)</td><td>1.98</td><td>1.30</td><td>1.05</td><td>1.30</td><td>-0.88</td><td>-0.25</td><td>1.98</td></tr></table>	*CO1*	*PO1*	**07**
			UNIT - 3			
	4	a)	Find the Fourier transform of $f(x) = e^{-a\|x\|}, a > 0$.	*CO1*	*PO1*	**06**
		b)	Solve $\int_0^\infty f(x) cosux\, dx = \begin{cases} 1-u & 0 < u < 1 \\ 0 & u \geq 1 \end{cases}$ and hence deduce that $\int_0^\infty \frac{sin^2 x}{x^2} dx = \frac{\pi}{2}$.	*CO1*	*PO1*	**07**
		c)	Apply Convolution theorem to find $F(f * g)$ where $f(x) = g(x) = \begin{cases} 1, & \|x\| \leq 1 \\ 0, & \|x\| > 1 \end{cases}$.	*CO1*	*PO1*	**07**
			UNIT - 4			
	5	a)	Derive Schmidt explicit formula for the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$.	*CO1*	*PO1*	**06**
		b)	Evaluate the pivotal values of the equation $u_{tt} = 16 u_{xx}$ under the conditions $u(0,t) = 0 = u(5,t), t \geq 0$, $u(x,0) = x^2(5-x)$ for $0 \leq x \leq 5$ and $u_t(x,0) = 0$ up to two-time levels, with $h = 1$ and $k = 1/4$.	*CO1*	*PO1*	**07**
		c)	Solve $u_{xx} = 32 u_t$ subject to the conditions $u(0,t) = 0$, $u(1,t) = t$ and $u(x,0) = 0$. Find the values of 'u' up to $t = 5$ by Bendre-Schmidt formula taking $h = 1/4$ and $k = 1$. Also find the values of (i) $u(0.75,4)$ (ii) $u(0.5,5)$.	*CO1*	*PO1*	**07**
			UNIT - 5			
	6	a)	Find the extremal of the functional $\int_{x_1}^{x_2} y' (x + y') dx$.	*CO1*	*PO1*	**06**
		b)	Derive the Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$.	*CO1*	*PO1*	**07**
		c)	Solve the difference equation $y_{n+2} - 3y_{n+1} + 2y_n = 0$ using Z-transform with $y_0 = 0$ and $y_1 = 1$.	*CO1*	*PO1*	**07**
			OR			
	7	a)	Find the Z-transform of $\sin(3n + 5)$.	*CO1*	*PO1*	**06**
		b)	Obtain the inverse Z-transform of $\frac{z(z+3)}{(z+1)(z-2)}$.	*CO1*	*PO1*	**07**
		c)	Show that the shape of a heavy cable that hangs freely under the gravity between two fixed points is a Catenary.	*CO1*	*PO1*	**07**

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