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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

April 2025 Semester End Make-Up Examinations

Programme: B.E. **Semester: III**
Course Code / Branch: **Duration: 3 hrs.**
23MA3BSTFN (Common to all branches except Civil Engg. And CS-Stream)
22MA3BSTFN (Common to all branches except CS-Stream) **Max Marks: 100**
Course: Transform Calculus, Fourier Series and Numerical Techniques

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

			CO	PO	Marks
		UNIT - 1			
1	a)	Find the Laplace transform of the following functions (i) $f(t) = te^{-3t}\cos(2t)$ (ii) $f(t) = \int_0^t \frac{1-e^{-t}}{t}dt$.	1	1	**6**
	b)	Express the given piecewise continuous function $f(t) = \begin{cases} 2t^2, & 0 \le t < 3 \\ t+4, & 3 \le t < 5 \\ 9, & t \ge 5 \end{cases}$ in terms of the unit step function and hence find its Laplace transform.	1	1	**7**
	c)	Apply Laplace transform techniques to solve the differential equation $\frac{d^2x}{dt^2} + 9x = \cos(2t)$ if $x(0) = 1$ and $x\left(\pi/2\right) = -1$.	1	1	**7**
		OR			
2	a)	Find the inverse Laplace transform of the following functions (i) $F(s) = \frac{s+3}{s^2-10s+29}$ (ii) $F(s) = \frac{1}{4s+1} + \frac{1}{5s-2}$.	1	1	**6**
	b)	If $f(t) = \begin{cases} 1+t, & 0 \le t < 1 \\ 3-t, & 1 \le t < 2 \end{cases}$ is a periodic function of period 2, then show that $L[f(t)] = \frac{1}{s} + \frac{1}{s^2}\tanh\left(\frac{s}{2}\right)$.	1	1	**7**
	c)	Using Laplace transform, solve the differential equation $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 4$ given $y(0) = 2$ and $y'(0) = 3$.	1	1	**7**
		UNIT - 2			
3	a)	Obtain the Fourier series for the periodic function $f(x)$ over the interval $(-\pi, \pi)$ where $f(x) = \begin{cases} x - \pi/2 & -\pi < x < 0 \\ x + \pi/2 & 0 < x < \pi \end{cases}$.	1	1	**6**
	b)	Obtain the Fourier series for $f(x) = e^{-ax},\ a > 0$, over the interval $(0, 2\pi)$ with $f(x + 2\pi) = f(x)$.	1	1	**7**

	c)	Find the complex Fourier series for the function $f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases}$ with $f(x+2\pi) = f(x)$ where k is a real constant, over the interval $(-\pi, \pi)$.	1	1	**7**
		OR			
4	a)	Obtain the Fourier series for the periodic function $f(x) = x\cos\left(\frac{\pi x}{l}\right)$ over $(-l, l)$.	1	1	**6**
	b)	Obtain the Fourier series of the periodic function $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$.	1	1	**7**
	c)	The following values of y give the displacement in inches of a certain machine part for the rotation x of the flywheel. Expand y in the form of a Fourier series up to the first harmonic. x: $0°$, $60°$, $120°$, $180°$, $240°$, $300°$ y: 0, 9.2, 14.4, 17.8, 17.3, 11.7	1	1	**7**
		UNIT - 3			
5	a)	Find the Fourier sine transform of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$.	1	1	**6**
	b)	Find the Fourier transform of $e^{-a^2x^2}$, $a > 0$.	1	1	**7**
	c)	Apply Fourier transform technique and solve the integral equation $\int_0^\infty f(x)\, cos\alpha x\, dx = \begin{cases} 1 - \alpha, & 0 \le \alpha \le 1 \\ 0, & \alpha > 1 \end{cases}$.	1	1	**7**
		OR			
6	a)	Find the Fourier cosine transform of the following funcitons (i) $f(x) = \begin{cases} x, & 0 < x < a \\ 0, & otherwise \end{cases}$ (ii) $f(x) = e^{-ax}$.	1	1	**6**
	b)	Find the Fourier transform of $f(x) = \begin{cases} 1 - \lvert x \rvert, & for\ \lvert x \rvert \le 1 \\ 0, & for\ \lvert x \rvert > 1 \end{cases}$.	1	1	**7**
	c)	By employing the convolution theorem, show that inverse Fourier transform of $e^{-\frac{s^2}{2}}$ is $\frac{1}{\sqrt{2\pi}} e^{-x^2/2}$. Given that $F\left[e^{-\frac{s^2}{4}}\right] = \frac{e^{-x^2}}{\sqrt{\pi}}$.	1	1	**7**
		UNIT - 4			
7	a)	Derive the finite difference formula to solve one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$.	1	1	**6**
	b)	Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ using Schmidt method with the conditions $u(x,0) = sin(\pi x), 0 \le x \le 1, u(0,t) = 0, u(1,t) = 0$. Carryout the computations for two-time levels taking $h = \frac{1}{3}$ and $k = \frac{1}{36}$.	1	1	**7**
	c)	Evaluate the pivotal values of the equation $u_{tt} = 16u_{xx}$ taking $h = 1$ and $k = 0.25$ up to $t = 0.5$. The boundary conditions are $u(0,t) = u(5,t) = 0$ and initial conditions $u(x,0) = x^2(5-x)$ and $\frac{\partial u}{\partial t}(x,0) = 0$.	1	1	**7**
		OR			

	8	a)	Derive the Bendre-Schmidt formula to solve numerically the one-dimensional heat equation $u_t = c^2 u_{xx}$.	1	1	**6**
		b)	Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ with the initial conditions $u(x,0) = sin(\pi x), 0 \le x \le 1,\ u_t(x,0) = 0, 0 \le x \le 1$ and the boundary conditions $u(0,t) = u(1,t) = 0$. Carryout the computations up to two time levels taking $h = 0.25$ and $k = 0.5$.	1	1	**7**
		c)	Find the numerical solution of the parabolic equation $3u_t = u_{xx}$ when $u(0,t) = 0, u(4,t) = 0$ and $u(x,0) = x(4-x)$ by taking $h = 1$ and $k = 1$. Carryout the computations up to two-time levels.	1	1	**7**
			UNIT - 5			
	9	a)	Show that a necessary condition for $I = \int_{x_1}^{x_2} f(x,y,y')dx$ to be an extremum is that $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$.	1	1	**6**
		b)	Find the extremal of the functional $\int_0^{\pi/2} (y^2 - y'^2 - 2y\sin x)dx$ with $y(0) = y\left(\frac{\pi}{2}\right) = 0$.	1	1	**7**
		c)	Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using Z-transform.	1	1	**7**
			OR			
	10	a)	Show that a heavy cable that hangs freely under gravity between two fixed points is in the shape of a catenary.	1	1	**6**
		b)	Find the extremal of the functional $\int_0^{\pi/2} \left(y'^2 - y^2 + 2xy\right)dx$ with $y(0) = y\left(\pi/2\right) = 0$.	1	1	**7**
		c)	Solve the difference equation $u_{n+2} - 3u_{n+1} + 2u_n = 0$ with $u_0 = 0$, $u_1 = 1$ using Z-transform.	1	1	**7**

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