U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

January / February 2025 Semester End Main Examinations

Programme: B.E. **Semester: III**

Course Code / Branch: **Duration: 3 hrs.**

23MA3BSTFN (Common to all branches except Civil Engg. And CS-Stream)

22MA3BSTFN (Common to all branches except CS-Stream) **Max Marks: 100**

Course: Transform Calculus, Fourier Series and Numerical Techniques

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	**CO**	**PO**	**Marks**
1	a)	Evaluate $(i)\ L\left[e^{-t}t\cos 3t\right]$ $(ii)\ L\left[t\int_0^t \frac{e^{-t}\sin t}{t}dt\right]$.	1	1	**6**
	b)	Obtain the Laplace transform of a square wave function of period $2a$ given by $f(t)=\begin{cases} k, & 0<t<a \\ -k, & a<t<2a \end{cases}$.	1	1	**7**
	c)	Solve the initial value problem $y''-2y'-8y=0,\ y(0)=3$ and $y'(0)=6$ by using Laplace transform.	1	1	**7**
		OR			
2	a)	Find the Laplace transform of $\left(\sqrt{t}+\frac{1}{\sqrt{t}}\right)^3$.	1	1	**6**
	b)	Evaluate (i) $L^{-1}\left\{\frac{3s+7}{s^2-2s-3}\right\}$ (ii) $L^{-1}\left\{\frac{1}{4s-1}+\frac{1}{s^2-25}\right\}$	1	1	**7**
	c)	Express the function $f(t)=\begin{cases} \sin t, & 0<t\le\pi \\ \sin 2t, & \pi<t\le 2\pi \\ \sin 3t, & t>2\pi \end{cases}$ in terms of unit step function and hence find its Laplace transform.	1	1	**7**
		UNIT - 2			
3	a)	Obtain the complex Fourier series of the function $f(x)=e^{ax}$ over the interval $(-\pi,\pi)$.	1	1	**6**
	b)	Obtain the Fourier series expansion for the periodic function $f(x)=\begin{cases} 1+\frac{4x}{3} & -\frac{3}{2}<x<0 \\ 1-\frac{4x}{3} & 0<x<\frac{3}{2} \end{cases}$.	1	1	**7**

<table>
<tr><td></td><td></td><td>c)</td><td>The following table gives the variations of a periodic current A over a period T. Show that there is a constant part of 0.75amp in the current A and obtain the amplitude of the first harmonic.
<table><tr><td>t(secs)</td><td>0</td><td>T/6</td><td>T/3</td><td>T/2</td><td>2T/3</td><td>5T/6</td><td>T</td></tr><tr><td>A(amp)</td><td>1.98</td><td>1.30</td><td>1.05</td><td>1.30</td><td>-0.88</td><td>-0.25</td><td>1.98</td></tr></table></td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>4</td><td>a)</td><td>Obtain the Fourier series expansion of $f(x)=\dfrac{(\pi-x)}{2}$ in $-\pi<x<\pi$ with $f(x+2\pi)=f\left(x\right)$.</td><td></td><td></td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Obtain the Fourier series expansion of the periodic function $f(x)=x\left(2l-x\right)$ in $\left(0,2l\right)$.</td><td></td><td></td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Expand y as a Fourier series up to the first harmonic using the given data
<table><tr><td>$x:$</td><td>0</td><td>60</td><td>120</td><td>180</td><td>240</td><td>300</td><td>360</td></tr><tr><td>$y:$</td><td>7.9</td><td>7.2</td><td>3.6</td><td>0.5</td><td>0.9</td><td>6.8</td><td>7.9</td></tr></table></td><td></td><td></td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 3</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Find the Fourier Transform of the function $f(x)=e^{-a|x|},\quad a>0.$</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find the Fourier sine transform of $f(x)=\dfrac{e^{-ax}}{x},\quad a>0.$</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Using Fourier transform technique, solve the integral equation $\int_0^\infty f\left(\theta\right)\cos\left(\alpha\theta\right)d\theta=\begin{cases}1-\alpha, & 0\le\alpha\le1\\ 0, & \alpha>1\end{cases}.$</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>6</td><td>a)</td><td>Obtain the Fourier cosine and sine transforms of $f(x)=e^{-ax}$.</td><td></td><td></td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find the inverse Fourier transform of $F(s)=e^{-s^2}$.</td><td></td><td></td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>By employing the convolution theorem, show that inverse Fourier transform of $e^{-\frac{s^2}{2}}$ is $\dfrac{1}{\sqrt{2\pi}}e^{-x^2/2}$. Given that $F\left[e^{-\frac{s^2}{4}}\right]=\dfrac{e^{-x^2}}{\sqrt{\pi}}$.</td><td></td><td></td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 4</td><td></td><td></td><td></td></tr>
<tr><td></td><td>7</td><td>a)</td><td>Derive the three level finite difference formula to solve numerically the one dimensional wave equation $u_{tt}=c^2u_{xx}$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find the numerical solution of the parabolic equation $u_t=u_{xx}$ under the conditions $u\left(0,t\right)=0,\ u\left(1,t\right)=0,\ t\ge0$ and $u\left(x,0\right)=\sin\pi x,\ 0\le x\le1$ by taking $h=1/4$ and $k=1/96$ using Schmidt explicit method up to two time levels.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Solve numerically $\dfrac{\partial^2u}{\partial t^2}=4\dfrac{\partial^2u}{\partial x^2}$ subject to the conditions $u\left(0,t\right)=0,\ u\left(4,t\right)=0,\ u_t\left(x,0\right)=0$ and $u\left(x,0\right)=x\left(4-x\right)$ by taking $h=1,\ k=0.5$ up to two time levels.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
</table>

	8	a)	Derive the Bendre-Schmidt formula to solve numerically the one-dimensional heat equation $u_t = c^2 u_{xx}$.			**6**
		b)	Solve the initial boundary value problem $u_t = u_{xx}$, at $t = 0.002$ under the conditions $u(0,t) = 0 = u(1,t)$ and $u(0,t) = f(x)$, $0 \le x \le 1$ using Schmidt method by taking $h = 0.1, k = 0.001$, where $f(x) = \begin{cases} 2x, & 0 \le x \le 1/2 \\ 2(1-x) & 1/2 \le x \le 1 \end{cases}$.			**7**
		c)	Find the numerical solution $u(x,t)$ of hyperbolic equation $u_{tt} = u_{xx}$ under the conditions $u(0,t) = 0, u(1,t) = 0$, $u_t(x,0) = 0$ and $u(x,0) = \sin \pi x$, $0 \le x \le 1$, by taking $h = 1/4$ and $k = 1/5$. Compute $u(x,t)$ up to two time levels.			**7**
			UNIT - 5			
	9	a)	A heavy cable hangs freely under gravity between two fixed points. Show that the shape of the cable is a Catenary.	1	1	**6**
		b)	Find the extremal of the functional $I = \int_{x_1}^{x_2} (y^2 + y'^2 + 2ye^x) dx$.	1	1	**7**
		c)	Solve $u_{n+2} + 2u_{n+1} + u_n = n$ with $u_0 = u_1 = 0$ using Z-transform.	1	1	**7**
			OR			
	10	a)	Derive the Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$.	1	1	**6**
		b)	Show that the extremal of $I = \int_{x_1}^{x_2} \sqrt{y(1 + y'^2)}\, dx$ is a parabola.	1	1	**7**
		c)	Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using Z-transform.	1	1	**7**

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