

U.S.N.

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

October 2024 Supplementary Examinations

Programme: B.E.

Semester: III

Course Code & Branch:

Duration: 3 hrs.

23MA3BSTFN (Common to all Branches except Civil Engg. & CS-Stream) /

Max Marks: 100

22MA3BSTFN (Common to all Branches except CS-Stream)

Course: Transform Calculus, Fourier Series and Numerical Techniques

- Instructions:** 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

| Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice. | | UNIT - 1 | CO | PO | Marks |
|--|---|--|-----|-----|-------|
| | 1 | a) Find the Laplace transform of i) $f(t) = te^{2t} \sin(3t)$ ii) $f(t) = \frac{\sin(3t)}{t}$. | CO1 | PO1 | 06 |
| | | b) Express the function $f(t) = \begin{cases} \cos t & 0 < t < \pi \\ \cos 2t & \pi < t < 2\pi \\ \cos 3t & t > 2\pi \end{cases}$ in terms of unit step function and hence find its Laplace transform. | CO1 | PO1 | 07 |
| | | c) Solve $y'' + 2y' - 3y = \sin(t)$ with $y(0) = 0 = y'(0)$ by the method of Laplace transform. | CO1 | PO1 | 07 |
| | | OR | | | |
| | 2 | a) Find the Laplace transform of $f(t) = t \int_0^t \frac{e^{-4t} \sin t}{t} dt$. | CO1 | PO1 | 06 |
| | | b) Show that the Laplace transform of the triangular wave function $f(x) = \begin{cases} a & 0 \leq t \leq a \\ 2a-t & a \leq t \leq 2a \end{cases}$ of period $2a$ is $\frac{1}{s^2} \tanh\left(\frac{as}{2}\right)$. | CO1 | PO1 | 07 |
| | | c) Find the inverse Laplace transform of i) $F(s) = \frac{s+5}{s^2-6s+13}$ ii) $F(s) = \frac{(s+1)^2}{(s+2)^4}$. | CO1 | PO1 | 07 |
| | | UNIT - 2 | | | |
| | 3 | a) Obtain the complex form of the Fourier series for the periodic function $f(x) = e^{-x}$ in $-1 \leq x \leq 1$. | CO1 | PO1 | 06 |
| | | b) Find the Fourier series of the periodic function $f(x) = x - x^2$ in the interval $(-\pi, \pi)$. | CO1 | PO1 | 07 |

| | | | | | | | | | | | | | | | | | | | | | | | |
|-----|----|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|---|-----|---|-----|---|-----|---|-----|-----|-----|----|
| | c) | Determine the constant term and the first harmonic term of the Fourier series expansion of y from the given data <table><tr><td>x</td><td>0</td><td>45</td><td>90</td><td>135</td><td>180</td><td>225</td><td>270</td><td>315</td></tr><tr><td>y</td><td>2</td><td>1.5</td><td>1</td><td>1.5</td><td>0</td><td>0.5</td><td>1</td><td>1.5</td></tr></table> | x | 0 | 45 | 90 | 135 | 180 | 225 | 270 | 315 | y | 2 | 1.5 | 1 | 1.5 | 0 | 0.5 | 1 | 1.5 | COI | POI | 07 |
| x | 0 | 45 | 90 | 135 | 180 | 225 | 270 | 315 | | | | | | | | | | | | | | | |
| y | 2 | 1.5 | 1 | 1.5 | 0 | 0.5 | 1 | 1.5 | | | | | | | | | | | | | | | |
| | | UNIT - 3 | | | | | | | | | | | | | | | | | | | | | |
| 4 | a) | Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2 & \text{for } x \leq a \\ 0 & \text{for } x > a \end{cases}$. | COI | POI | | | | 06 | | | | | | | | | | | | | | | |
| | b) | Find the inverse Fourier sine transform of $\frac{e^{-as}}{s}$, $a > 0$. | COI | POI | | | | 07 | | | | | | | | | | | | | | | |
| | c) | Obtain the Fourier cosine transform of e^{-ax} , $a > 0$ and hence deduce the Fourier cosine transform of xe^{-ax} and also evaluate $\int_0^\infty \frac{\cos(\alpha x)}{a^2 + \alpha^2} d\alpha$. | COI | POI | | | | 07 | | | | | | | | | | | | | | | |
| | | UNIT - 4 | | | | | | | | | | | | | | | | | | | | | |
| 5 | a) | Derive Bendre-Schmidt formula for the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$. | COI | POI | | | | 06 | | | | | | | | | | | | | | | |
| | b) | Solve numerically the wave equation $u_{tt} = 16u_{xx}$ subject to the boundary conditions $u(0,t) = 0 = u(5,t)$, $t > 0$ and the initial conditions $u_t(x,0) = 0$, $u(x,0) = x^2(5-x)$ taking $h = 1$, $k = 0.25$ up to two time levels. | COI | POI | | | | 07 | | | | | | | | | | | | | | | |
| | c) | Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1$ at $t = 0.001$ under the conditions $u(0,t) = 0 = u(1,t)$, $u(x,0) = f(x)$ where $f(x) = \begin{cases} 2x, & 0 \leq x \leq 0.5 \\ 2(1-x), & 0.5 \leq x \leq 1 \end{cases}$ by taking $h = 0.1$, $k = 0.001$. | COI | POI | | | | 07 | | | | | | | | | | | | | | | |
| | | UNIT - 5 | | | | | | | | | | | | | | | | | | | | | |
| 6 | a) | Find the Z-transform of $\sin(3n+5)$. | COI | POI | | | | 06 | | | | | | | | | | | | | | | |
| | b) | Solve $y_{n+1} + 2y_{2n+1} + y_n = n$ with $y_0 = 0 = y_1$ by using Z-transform. | COI | POI | | | | 07 | | | | | | | | | | | | | | | |
| | c) | Find the path on which a particle in the absence of friction, will slide from one point to another in the shortest time under the action of gravity. | COI | POI | | | | 07 | | | | | | | | | | | | | | | |
| | | OR | | | | | | | | | | | | | | | | | | | | | |
| 7 | a) | Derive the Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$. | COI | POI | | | | 06 | | | | | | | | | | | | | | | |
| | b) | A heavy cable hangs freely under the gravity between two fixed points. Show that the shape of the cable is a Catenary. | COI | POI | | | | 07 | | | | | | | | | | | | | | | |
| | c) | Find the inverse Z-transform of $\frac{z^3 - 20z}{(z-2)^3(z-4)}$. | COI | POI | | | | 07 | | | | | | | | | | | | | | | |
