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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

October 2024 Supplementary Examinations

Programme: B.E.

Semester: III

Course Code & Branch:

Duration: 3 hrs.

23MA3BSTFN (Common to all Branches except Civil Engg. & CS-Stream) /

Max Marks: 100

22MA3BSTFN (Common to all Branches except CS-Stream)

Course: Transform Calculus, Fourier Series and Numerical Techniques

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

UNIT - 1			CO	PO	Marks
1	a)	Find the Laplace transform of i) $f(t) = te^{2t} \sin(3t)$ ii) $f(t) = \frac{\sin(3t)}{t}$.	CO1	PO1	06
	b)	Express the function $f(t) = \begin{cases} \cos t & 0 < t < \pi \\ \cos 2t & \pi < t < 2\pi \\ \cos 3t & t > 2\pi \end{cases}$ in terms of unit step function and hence find its Laplace transform.	CO1	PO1	07
	c)	Solve $y'' + 2y' - 3y = \sin(t)$ with $y(0) = 0 = y'(0)$ by the method of Laplace transform.	CO1	PO1	07
OR					
2	a)	Find the Laplace transform of $f(t) = t \int_0^t \frac{e^{-4t} \sin t}{t} dt$.	CO1	PO1	06
	b)	Show that the Laplace transform of the triangular wave function $f(x) = \begin{cases} a & 0 \leq t \leq a \\ 2a-t & a \leq t \leq 2a \end{cases}$ of period $2a$ is $\frac{1}{s^2} \tanh\left(\frac{as}{2}\right)$.	CO1	PO1	07
	c)	Find the inverse Laplace transform of i) $F(s) = \frac{s+5}{s^2 - 6s + 13}$ ii) $F(s) = \frac{(s+1)^2}{(s+2)^4}$.	CO1	PO1	07
UNIT - 2					
3	a)	Obtain the complex form of the Fourier series for the periodic function $f(x) = e^{-x}$ in $-1 \leq x \leq 1$.	CO1	PO1	06
	b)	Find the Fourier series of the periodic function $f(x) = x - x^2$ in the interval $(-\pi, \pi)$.	CO1	PO1	07

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

	c)	Determine the constant term and the first harmonic term of the Fourier series expansion of y from the given data <table border="1"> <tr> <td>x</td><td>0</td><td>45</td><td>90</td><td>135</td><td>180</td><td>225</td><td>270</td><td>315</td></tr> <tr> <td>y</td><td>2</td><td>1.5</td><td>1</td><td>1.5</td><td>0</td><td>0.5</td><td>1</td><td>1.5</td></tr> </table>	x	0	45	90	135	180	225	270	315	y	2	1.5	1	1.5	0	0.5	1	1.5	CO1	PO1	07
x	0	45	90	135	180	225	270	315															
y	2	1.5	1	1.5	0	0.5	1	1.5															
		UNIT - 3																					
4	a)	Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2 & \text{for } x \leq a \\ 0 & \text{for } x > a \end{cases}$.	CO1	PO1	06																		
	b)	Find the inverse Fourier sine transform of $\frac{e^{-as}}{s}$, $a > 0$.	CO1	PO1	07																		
	c)	Obtain the Fourier cosine transform of e^{-ax} , $a > 0$ and hence deduce the Fourier cosine transform of xe^{-ax} and also evaluate $\int_0^{\infty} \frac{\cos(\alpha x)}{a^2 + \alpha^2} d\alpha$.	CO1	PO1	07																		
		UNIT - 4																					
5	a)	Derive Bremde-Schmidt formula for the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$.	CO1	PO1	06																		
	b)	Solve numerically the wave equation $u_{tt} = 16u_{xx}$ subject to the boundary conditions $u(0,t) = 0 = u(5,t)$, $t > 0$ and the initial conditions $u_t(x,0) = 0$, $u(x,0) = x^2(5-x)$ taking $h = 1$, $k = 0.25$ up to two time levels.	CO1	PO1	07																		
	c)	Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1$ at $t = 0.001$ under the conditions $u(0,t) = 0 = u(1,t)$, $u(x,0) = f(x)$ where $f(x) = \begin{cases} 2x, & 0 \leq x \leq 0.5 \\ 2(1-x), & 0.5 \leq x \leq 1 \end{cases}$ by taking $h = 0.1$, $k = 0.001$.	CO1	PO1	07																		
		UNIT - 5																					
6	a)	Find the Z-transform of $\sin(3n + 5)$.	CO1	PO1	06																		
	b)	Solve $y_{n+1} + 2y_{2n+1} + y_n = n$ with $y_0 = 0 = y_1$ by using Z-transform.	CO1	PO1	07																		
	c)	Find the path on which a particle in the absence of friction, will slide from one point to another in the shortest time under the action of gravity.	CO1	PO1	07																		
		OR																					
7	a)	Derive the Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$.	CO1	PO1	06																		
	b)	A heavy cable hangs freely under the gravity between two fixed points. Show that the shape of the cable is a Catenary.	CO1	PO1	07																		
	c)	Find the inverse Z-transform of $\frac{z^3 - 20z}{(z-2)^3(z-4)}$.	CO1	PO1	07																		
