U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

June 2025 Semester End Main Examinations

Programme: B.E. **Semester: III**
Course Code / Branch: **Duration: 3 hrs.**
23MA3BSTFN (Common to all branches except Civil Engg. And CS-Stream)
22MA3BSTFN (Common to all branches except CS-Stream) **Max Marks: 100**
Course: Transform Calculus, Fourier Series and Numerical Techniques

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.

		UNIT - 1	*CO*	*PO*	**Marks**
1	a)	Obtain the Laplace transform of the function $f(t) = e^{-t}\left(3+\sqrt{t}\right) + t\sin t$.	1	1	**6**
	b)	Obtain the Laplace transform of a periodic function $f(t)$ with a period 2 defined by $f(t) = \begin{cases} 2t, & 0 \le t < 1 \\ 4, & 1 \le t < 2 \end{cases}$.	1	1	**7**
	c)	Solve the differential equation $4y'' + 24y' + 20y = 200$, $y(0) = 0,\ y'(0) = 0$ using Laplace transform method.	1	1	**7**
		OR			
2	a)	Evaluate (i) $L\left(e^{-t}\int_0^t \frac{\sin t}{t}dt\right)$ (ii) $L\left[t\sin^2 t\right]$	1	1	**6**
	b)	Express the function $f(t) = \begin{cases} t, & 0 \le t < 1 \\ 2-t, & 1 \le t < 2 \\ 0, & t \ge 2 \end{cases}$ in terms of the unit step function and hence obtain its Laplace transform.	1	1	**7**
	c)	Obtain the inverse Laplace transform of the following functions. (i) $F(s) = \frac{s-1}{s^2+6s+8}$ (ii) $F(s) = \frac{3}{(s+2)(s+3)}$.	1	1	**7**
		UNIT - 2			
3	a)	Find the Fourier series of the function $f(x) = x^2$ over the interval $(-\pi, \pi)$.	1	1	**6**
	b)	Obtain the Fourier series of a square wave function given by $f(x) = \begin{cases} k, & -1 < x < 0 \\ -k, & 0 < x < 1 \end{cases}$ with $f(x+2) = f(x)$.	1	1	**7**

<table>
<tr><td></td><td></td><td>c)</td><td>Obtain the constant term and the coefficients of the first cosine and sine terms in the Fourier expansion of y from the table:
<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>9</td><td>18</td><td>24</td><td>28</td><td>26</td><td>20</td></tr></table></td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>4</td><td>a)</td><td>Find the complex form of Fourier series of $f(x) = e^{-x}$ in the interval $(-1, 1)$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find the Fourier series expansion of a periodic function
$f(x) = \begin{cases} \pi - x & 0 \leq x \leq \pi \\ x - \pi & \pi \leq x \leq 2\pi \end{cases}$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Obtain the Fourier series of y up to the first harmonic and amplitude of the first harmonic for the following data:
<table><tr><td>x</td><td>45</td><td>90</td><td>135</td><td>180</td><td>225</td><td>270</td><td>315</td><td>360</td></tr><tr><td>y</td><td>4</td><td>3.8</td><td>2.4</td><td>2</td><td>-1.5</td><td>0</td><td>2.8</td><td>3.4</td></tr></table></td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 3</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Obtain the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & for\ |x| \leq 1 \\ 0, & for\ |x| > 1 \end{cases}$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find the Fourier sine transform of $f(x) = \frac{e^{-ax}}{x}$ $a > 0$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Using Fourier transform techniques, solve the integral equation
$\int_0^\infty f(x) cos\alpha x\, dx = \begin{cases} 1 - \alpha, & 0 \leq \alpha \leq 1 \\ 0, & \alpha > 1 \end{cases}$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>6</td><td>a)</td><td>Obtain the Fourier cosine transform of $f(x) = e^{-ax} \cos ax$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find the inverse Fourier transform of $F(s) = e^{-s^2}$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Use convolution theorem to find the inverse Fourier transform of $\frac{1}{(1+s^2)^2}$, given that $\frac{2}{(1+s^2)}$ is the Fourier transform of $e^{-|x|}$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 4</td><td></td><td></td><td></td></tr>
<tr><td></td><td>7</td><td>a)</td><td>Derive the Bendre-Schmidt formula to solve numerically the one-dimensional heat equation $u_t = c^2 u_{xx}$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ using Schmidt method with the initial condition $u(x, 0) = \sin(\pi x), 0 \leq x \leq 1$ and the boundary conditions $u(0, t) = 0,\ u(1, t) = 0$. Carryout the computations for two-time levels taking $h = \frac{1}{4}$ and $k = \frac{1}{48}$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>By using the three-level formula, solve the hyperbolic equation $u_{tt} = 4u_{xx}$ subject to the boundary conditions $u(0, t) = 0$, $u(4, t) = 0,\ t \geq 0$ and the initial conditions $\frac{\partial u}{\partial t}(x, 0) = 0$, $u(x, 0) = x(4 - x), 0 \leq x \leq 4$. Carryout the computations up to two time levels taking h = 1 and k = 1/2.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
</table>

	8	a)	Derive the three level finite difference formula to solve numerically the one dimensional wave equation $u_{tt} = c^2 u_{xx}$.	*1*	*1*	**6**
		b)	Apply Bendre-Schmidt formula to solve $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$ using with the initial condition $u(x,0) = x(4-x), 0 < x < 4,$ and the boundary conditions $u(0,t) = u(4,t) = 0, t \geq 0$. Carryout the computations up to two time levels taking $h = 1$.	*1*	*1*	**7**
		c)	Evaluate the pivotal values of the equation $u_{tt} = 16u_{xx}$ taking $h = 1$ and $k = 0.25$ up to $t = 0.5$. The boundary conditions are $u(0,t) = u(5,t) = 0$ and initial conditions $u(x,0) = x^2(5-x)$ and $\frac{\partial u}{\partial t}(x,0) = 0$.	*1*	*1*	**7**
			UNIT - 5			
	9	a)	Find the path on which a particle in the absence of friction, will slide from one point to another in the shortest time under the action of gravity.	*1*	*1*	**6**
		b)	Show that the curve joining the points $(1,0)$ and $(2,1)$ for which $I = \int_1^2 \frac{1}{x}\sqrt{1+y'^2}\,dx$ is an extremum is a circle.	*1*	*1*	**7**
		c)	Solve the difference equation $u_{n+2} - 2u_{n+1} + u_n = 2^n$ with $u_0 = 2$, $u_1 = 1$.	*1*	*1*	**7**
			OR			
	10	a)	Show that the necessary condition for $I = \int_{x_1}^{x_2} f(x,y,y')dx$ to be an extremum is that $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$.	*1*	*1*	**6**
		b)	Discuss the Hanging cable problem.	*1*	*1*	**7**
		c)	Solve the difference equation $y_{n+2} - 4y_{n+1} + 4y_n = 0$ with $y_0 = 1,\ y_1 = 0$.	*1*	*1*	**7**

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