U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

June 2025 Semester End Main Examinations

Programme: B.E. **Semester: IV**
Branch: Computer Science and Business Systems **Duration: 3 hrs.**
Course Code: 24MA4BSABS **Max Marks: 100**
Course: Foundations of Algebra for Business Systems

Instructions: 1. Each unit has an internal choice; answer one complete question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	CO	PO	Marks
1	a)	Show that the set of all integers $\mathbb{Z}$ is a group with respect to addition.	1	1	**6**
	b)	Show that the Klein-4 group $A=\{e, a,b,c\}$ where e is identity element is an abelian group.	1	1	**7**
	c)	Prove that the set $\{0,1,2,3......, p-1\}(\text{mod } p)$, where p is a prime, is a field with respect to operations addition$(\text{mod } p)$ and multiplication$(\text{mod } p)$.	1	1	**7**
		OR			
2	a)	Define sub group of a group. Let $H=\{0, 2, 4\}$. Show that $(H,+)$ is a subgroup of $(Z_6, +)$.	1	1	**6**
	b)	Define abelian group and show that fourth root of unity is an abelian group.	1	1	**7**
	c)	Prove that the set of integers $\mathbb{Z}$ is a commutative ring in which addition and multiplication are defined as $a\oplus b=a+b-1$ and $a\odot b=a+b-ab$ where $a,b\in\mathbb{Z}$.	1	1	**7**
		UNIT - 2			
3	a)	Consider the matrix $A=\begin{bmatrix}1 & -2 & 1\\ 3 & -1 & 0\\ 1 & 4 & -2\end{bmatrix}$ which defines a linear operator on $\mathbb{R}^3$. Find the matrix of the linear transformation relative to the basis $S=\{u_1,u_2,u_3\}=\left\{\begin{bmatrix}1\\1\\1\end{bmatrix},\begin{bmatrix}0\\1\\1\end{bmatrix},\begin{bmatrix}1\\2\\3\end{bmatrix}\right\}$.	1	1	**6**

		b)	Find and sketch the images of the unit square and the unit circle under the affine transformation $T(u) = Au + v$ defined by the matrix $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ and the vector $v = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.	1	1	**7**
		c)	Find the change-of-basis matrix P from S to E in $\mathbb{R}^3$ when $E = \{(1,0,0),(0,1,0),(0,0,1)\}$ and $S = \{(1, 0, 1), (2, 1, 2), (1, 2, 2)\}$. Hence find change-of-basis matrix Q from E to S.	1	1	**7**
			OR			
	4	a)	Discuss the following maps on $\mathbb{R}^2$ and represent them graphically: i. Horizontal contraction or expansion. ii. Vertical contraction or expansion. iii. Vertical shear.	1	1	**6**
		b)	Given $G: \mathbb{R}^2 \to \mathbb{R}^2$ and $S = \{u_1, u_2\} = \{(1, 3), (2, 5)\}$ is the basis of $\mathbb{R}^2$ and $G(x, y) = (2x - 7y, 4x + 3y)$, verify $[G]_s . [v]_s = [G(v)]_s$ for the vector $v = (4, -3)$ in $\mathbb{R}^2$.	1	1	**7**
		c)	Is T the linear operator on $\mathbb{R}^3$ defined by $T(x, y, z) = (2x,\ 4x - y\ ,\ 2x + 3y - z)$ invertible.? If so, find T^{-1} and T^{-2}.	1	1	**7**
			UNIT - 3			
	5	a)	Consider $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & 4 \\ 1 & 4 & 6 \end{bmatrix}$ and the vectors $u = \begin{bmatrix} 1 \\ \alpha \\ 1 \end{bmatrix}$ and $v = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$. The inner product $\langle u, v \rangle = u^T A v$. Find the value of α if it is known that the vectors u and v are orthogonal and hence find the length of u relative to the given inner product.	1	1	**6**
		b)	Let $P_2(t)$ is the vector space of polynomials of degree ≤ 2 with $\langle f, g \rangle = \int_0^1 f(t) g(t) dt$. Find a basis of the subspace W orthogonal to $h(t) = 2t + 1$.	1	1	**7**
		c)	Solve the system of equations $AX = B$ where $A = \begin{bmatrix} 2 & 3 \\ 2 & 4 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix}$ by QR factorization method.	1	1	**7**
			OR			

	6	a)	Find the angle between the vectors $A=\begin{bmatrix}9&8&7\\6&5&4\end{bmatrix}$ and $B=\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}$ where $\langle A,B\rangle = Tr\left(B^T A\right)$.	1	1	**6**
		b)	Apply the Gram-Schmidt Orthogonalization to the basis vectors $v_1=(1,0,0)$, $v_2=(0,1,0)$ and $v_3=(0,0,1)$ equipped with the inner product defined by $\langle x \quad y\rangle = 2x_1y_1+2x_2y_2+x_3y_3-x_2y_3-x_3y_2$.	1	1	**7**
		c)	Find the least square parabola of the form $y=a+bx+cx^2$ for the data (1, 1), (2, 4), (3, 7) and (4, 5).	1	1	**7**
			UNIT – 4			
	7	a)	Apply Cayley-Hamilton Theorem to compute A^4 given $A=\begin{bmatrix}1&2&3\\2&4&5\\3&5&6\end{bmatrix}$.	1	1	**6**
		b)	Find the eigenspaces of the linear transformation $T: P_2(t)\to P_2(t)$ defined by $T\big(f(t)\big)=(2a-c)t^2+(2a+b-2c)t+(-a+2c)$ where $f(t)=at^2+bt+c$.	1	1	**7**
		c)	Find the characteristic and minimal polynomials of $\begin{bmatrix}2&5&0&0&0\\0&2&0&0&0\\0&0&4&2&0\\0&0&3&5&0\\0&0&0&0&7\end{bmatrix}$.	1	1	**7**
			OR			
	8	a)	Apply Cayley-Hamilton Theorem to compute A^{-2} and A^{-3} if $A=\begin{bmatrix}4&6&6\\1&3&2\\-1&-4&-3\end{bmatrix}$.	1	1	**6**
		b)	Obtain the algebraic and geometric multiplicity of the eigenvalues of the matrix $A=\begin{bmatrix}2&0&1\\0&1&0\\1&0&2\end{bmatrix}$ by finding the eigenspaces.	1	1	**7**
		c)	Find the Jordan canonical form of $\begin{bmatrix}1&2&-1\\0&2&0\\1&-2&3\end{bmatrix}$.	1	1	**7**
			UNIT - 5			
	9	a)	Find nature of the quadratic form $x^2+5y^2+z^2+2xy+6xz+2yz$ and hence write its canonical form.	1	1	**10**
		b)	Determine a singular value decomposition of A= $\begin{bmatrix}3&2\\2&3\\2&-2\end{bmatrix}$.	1	1	**10**
			OR			

	10	a)	Orthogonally diagonalize $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$.	*1*	*1*	**10**
		b)	Reduce the dimension of the data given below from 2 to 1 using principal component analysis. X: 4, 8, 13, 7 Y: 11, 4, 5, 14	*1*	*1*	**10**

X	4	8	13	7
Y	11	4	5	14

******