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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

September / October 2023 Semester End Main Examinations

Programme: B.E.

Branch: AS/ME /EEE/ECE/ET/MD/CIVIL/EIE

Course Code: 22MA4BSCPS

Course: Complex Analysis, Probability and Statistical Methods

Semester: IV

Duration: 3 hrs.

Max Marks: 100

Date: 20.09.2023

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
 2. Missing data, if any, may be suitably assumed.
 3. Statistical tables are permitted.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

UNIT - I			CO	PO	Marks
1	a)	Show that $u(x, y) = 3xy^2 - x^3$ is harmonic and also find its analytic function $f(z)$.	CO1	PO1	6
	b)	Discuss the transformation $w = z + \frac{k^2}{z}$, ($z \neq 0$).	CO1	PO1	7
	c)	Verify the Cauchy's theorem for the function $f(z) = z^2$ taken over the boundary of the rectangle having vertices $(-1, 0), (1, 0), (1, 1)$ and $(-1, 1)$.	CO1	PO1	7
OR					
2	a)	Obtain the orthogonal trajectories of the family of curves $\left(r - \frac{1}{r}\right) \sin \theta = c$, $r \neq 0$.	CO1	PO1	6
	b)	If $f(z)$ is a regular function, then prove that $\left\{ \frac{\partial}{\partial x} f(z) \right\}^2 + \left\{ \frac{\partial}{\partial y} f(z) \right\}^2 = f'(z) ^2$.	CO1	PO1	7
	c)	Apply Cauchy's integral formula to evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$ where C is $ z =2$.	CO1	PO1	7
UNIT - II					
3	a)	Express $x^3 + 2x^2 - 4x + 5$ in terms of Legendre polynomial.	CO1	PO1	6
	b)	Obtain the series solution of Legendre's differential equation.	CO1	PO1	7
	c)	Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin(x)$ and $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos(x)$.	CO1	PO1	7

UNIT - III																							
4	a)	Obtain the rank correlation for the following data:	<i>CO1</i>	<i>PO1</i>	6																		
		<table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td><i>x</i></td><td>78</td><td>89</td><td>97</td><td>69</td><td>59</td><td>79</td><td>68</td><td>57</td></tr> <tr><td><i>y</i></td><td>125</td><td>137</td><td>156</td><td>112</td><td>107</td><td>138</td><td>123</td><td>108</td></tr> </table>	<i>x</i>	78	89	97	69	59	79	68	57	<i>y</i>	125	137	156	112	107	138	123	108			
<i>x</i>	78	89	97	69	59	79	68	57															
<i>y</i>	125	137	156	112	107	138	123	108															
	b)	If the co-efficient of correlation between two variables x and y is 0.5 and the acute angle between their lines of regression is $\tan^{-1}\left(\frac{3}{5}\right)$ then show that $\frac{\sigma_x}{\sigma_y} = 2$ or $\frac{1}{2}$.	<i>CO1</i>	<i>PO1</i>	7																		
	c)	<p>Fit a power function $y = ax^b$ to the following data pertaining to the demand for a product and its price charged at five different cities.</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>Price (Rs) <i>x</i></td><td>20</td><td>16</td><td>10</td><td>11</td><td>14</td></tr> <tr><td>Demand <i>y</i></td><td>22</td><td>41</td><td>120</td><td>89</td><td>56</td></tr> </table> <p>Predict the demand when price of the product is Rs. 12.</p>	Price (Rs) <i>x</i>	20	16	10	11	14	Demand <i>y</i>	22	41	120	89	56	<i>CO2</i>	<i>PO1</i>	7						
Price (Rs) <i>x</i>	20	16	10	11	14																		
Demand <i>y</i>	22	41	120	89	56																		
UNIT - IV																							
5	a)	<p>The number of accidents per day (<i>x</i>) as recorded in a garment factory over a period of 400 days is given.</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td><i>x</i></td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr> <tr><td><i>f</i></td><td>173</td><td>168</td><td>37</td><td>18</td><td>3</td><td>1</td></tr> </table> <p>Fit a Poisson distribution for the data and calculate the theoretical frequencies.</p>	<i>x</i>	0	1	2	3	4	5	<i>f</i>	173	168	37	18	3	1	<i>CO2</i>	<i>PO1</i>	6				
<i>x</i>	0	1	2	3	4	5																	
<i>f</i>	173	168	37	18	3	1																	
	b)	The mean inside diameter of a sample of 200 washers produced by a machine is 5.02 mm and the S.D. is 0.05 mm. The purpose for which these washers are intended allows a maximum tolerance in the diameter of 4.96 mm to 5.08 mm, otherwise the washers are considered to be defective. Determine the percentage of defective washers produced by the machine, assuming the diameters are normally distributed.	<i>CO1</i>	<i>PO1</i>	7																		
	c)	A coin is tossed three times. Let X denote 0 or 1 according as head or tail occurs on the first toss. Let Y denote the total number of heads which occur. Determine i) the marginal distributions of X and Y ii) the joint distributions of X and Y iii) $\text{Cov}(X, Y)$.	<i>CO1</i>	<i>PO1</i>	7																		
UNIT - V																							
6	a)	Mice with an average lifespan of 32 months will live up to 40 months when fed by a certain nutritious food. If 64 mice fed on this diet have an average lifespan of 38 months and standard deviation of 5.8 months. Test at 1% level of significance, is there any reason to believe that average lifespan is less than 40 months?	<i>CO2</i>	<i>PO1</i>	6																		

	b)	<p>In a Mathematics examination, 9 students of class A and 6 students of class B obtained the following marks. Test at 1% level of significance whether the average performance in mathematics is same or not for the two classes A and B. Assume that the samples are drawn from normal populations having same variance.</p> <table border="1"> <tr> <td>A</td><td>44</td><td>71</td><td>63</td><td>59</td><td>68</td><td>46</td><td>69</td><td>54</td><td>48</td></tr> <tr> <td>B</td><td>52</td><td>70</td><td>41</td><td>62</td><td>36</td><td>50</td><td></td><td></td><td></td></tr> </table>	A	44	71	63	59	68	46	69	54	48	B	52	70	41	62	36	50				CO2	PO1	7
A	44	71	63	59	68	46	69	54	48																
B	52	70	41	62	36	50																			
	c)	<p>The average weekly losses of man-hours due to strikes in an institute before and after a disciplinary program was implemented are as follows: Is there reason to believe that the disciplinary program is effective at 5% level of significance?</p> <table border="1"> <tr> <td>Before</td><td>45</td><td>73</td><td>46</td><td>124</td><td>33</td><td>57</td></tr> <tr> <td>After</td><td>36</td><td>60</td><td>44</td><td>119</td><td>35</td><td>51</td></tr> </table>	Before	45	73	46	124	33	57	After	36	60	44	119	35	51	CO2	PO1	7						
Before	45	73	46	124	33	57																			
After	36	60	44	119	35	51																			
		OR																							
7	a)	<p>If a random sample data show that 42 men earn on the average 744.85 with standard deviation 397.7, while 32 women earn on the average 516.78 with standard deviation 162.523. Test at 0.05 level of significance whether the average income for men and women is same or not.</p>	CO2	PO1	6																				
	b)	<p>An auditor claims that he takes on an average 10.5 days to file income tax returns (I.T. returns). Can this claim be accepted if a random sample shows that he took 13, 19, 15, 10, 12, 11, 14, 18 days to file I.T. returns at 0.01 level of significance?</p>	CO2	PO1	7																				
	c)	<p>Genetic theory states that children having one parent of blood type M and other blood type N will always be one of three types M, MN, N and that the proportions of these types will on average be 1: 2: 1. A report states that out of 300 children having one parent M and one N parent 30% were found to be of type M, 45% of type MN and remainder of type N. At 5% level of significance, whether the results are in the stated ratio or not.</p>	CO2	PO1	7																				
