U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

February 2025 Semester End Main Examinations

Programme: B.E. **Semester: IV**
Branch and Course Code: **Duration: 3 hrs.**
23MA4BSCPS (AS/ME/ECE/ETE/EIE) **Max Marks: 100**
22MA4BSCPS (AS/ME/EEE/ECE/ETE/MD/CIVIL/EIE)
Course: Complex Analysis, Probability and Statistical Methods

Instructions:
1. All units have internal choices, answer one complete question from each unit.
2. Missing data, if any, may be suitably assumed.
3. Use of Statistical tables is permitted.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	CO	PO	Marks
1	a)	Derive Cauchy-Riemann equations in polar form.	*1*	*1*	**6**
	b)	Construct the analytic function $f(z)=u(x,y)+iv(x,y)$ whose real part is $\frac{y}{x^2+y^2}$ by Milne-Thomson method and hence find its imaginary part.	*1*	*1*	**7**
	c)	Verify Cauchy's theorem for the integral of $\frac{1}{z}$ taken over the boundary of the triangle having vertices (1,2), (1,4) and (3,2).	*1*	*1*	**7**
		OR			
2	a)	Show that $u=x^3-3xy^2+3x^2-3y^2+1$ is harmonic and find its harmonic conjugate by constructing the analytic function $f(z)=u+iv$ using Milne-Thomson method.	*1*	*1*	**6**
	b)	Show that the transformation $w=z^2$ transforms the lines parallel to the coordinate axes in *z*-plane to the parabolic curves in *w*-plane.	*1*	*1*	**7**
	c)	Apply Cauchy's integral formula to evaluate $\int_C \frac{z+4}{z^2+2z+5}dz$ where *C* is $\lvert z+1-i\rvert=2$.	*1*	*1*	**7**
		UNIT - 2			
3	a)	If $x^3+2x^2-x+1=aP_0(x)+bP_1(x)+cP_2(x)+dP_3(x)$, then find the values of *a, b, c* and *d.*	*1*	*1*	**6**
	b)	Prove that (i) $J_{1/2}(x)=\sqrt{\frac{2}{\pi x}}\sin x$ (ii) $J_{-1/2}(x)=\sqrt{\frac{2}{\pi x}}\cos x$.	*1*	*1*	**7**
	c)	Obtain the series solution of the Legendre's differential equation.	*1*	*1*	**7**
		OR			
4	a)	Prove that (i) $J_{-n}(x)=(-1)^nJ_n(x)$ (ii) $J_n(-x)=(-1)^nJ_n(x)$ where n is a positive integer.	*1*	*1*	**6**

	b)	Derive the generating function for $J_n(x)$ in the form $e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$	*1*	*1*	**7**
	c)	Obtain the series solution of Bessel's differential equation.	*1*	*1*	**7**
		UNIT - 3			
5	a)	Compute the coefficient of rank correlation between industrial production and export using the following data and comment on the result. Production (in Crore tons): 55, 56, 58, 59, 60, 61, 62 Export (in Crore tons): 35, 38, 37, 39, 44, 43, 45	*1*	*1*	**6**
	b)	In a partially destroyed laboratory record, only the lines of regression of y on x and x on y are available as $4x-5y+33=0$ and $20x-9y=107$ respectively. Calculate $\bar{x}$, $\bar{y}$ and the coefficient of correlation between x and y.	*1*	*1*	**7**
	c)	If P is the pull required to lift a load W by means of a pulley block, find a linear law of the form $P=a+bW$ connecting P and W using the following data: P: 12, 15, 21, 25 W: 50, 70, 100, 120	*1*	*1*	**7**
		OR			
6	a)	Compute the coefficient of correlation for the following data: x: 21, 23, 30, 54, 57, 58 y: 60, 71, 72, 83, 110, 84	*1*	*1*	**6**
	b)	Fit a geometric curve $y=ax^b$ to the following data: x: 1, 2, 3, 4, 5 y: 0.5, 2, 4.5, 8, 12.5	*1*	*1*	**7**
	c)	The following results were obtained from the records of age (x) and blood pressure (y) of a group of 10 men. Mean: 53, 142 Variance: 130, 165 and $\sum(x-\bar{x})(y-\bar{y})=1220$. Find the appropriate regression equation and use it to estimate the blood pressure of a man whose age is 45.	*1*	*1*	**7**
		UNIT - 4			
7	a)	A joint probability distribution is given by the following table: Y \ X: -3, 2, 4 1: 0.1, 0.2, 0.2 3: 0.3, 0.1, 0.1 Find (i)The marginal distributions (ii) $\mathrm{Cov}(X,Y)$.	*1*	*1*	**6**

	b)	A source of liquid is known to contain bacteria with the mean number of bacteria per cubic centimeter equal to 3. Ten 1cc test tubes are filled with the liquid. Assuming that Poisson distribution is applicable, calculate the probability that all the test tubes will show growth i.e. contains at least one bacterium each.	*1*	*1*	**7**
	c)	In a normal distribution, 31% of the items are under 45 and 8% are over 64. Obtain the mean and standard deviation of the distribution.	*1*	*1*	**7**
		OR			
8	a)	Derive an expression for mean and variance of Poisson distribution.	*1*	*1*	**6**
	b)	A manufacturer of air-mail envelopes knows from experience that weight of the envelopes is normally distributed with mean 1.95 gm and standard deviation 0.05 gm. About how many envelopes weighing (i) 2 gm or more (ii) 2.05 gm or more can be expected in a given packet of 100 envelopes.	*1*	*1*	**7**
	c)	The distributions of two independent random variables X and Y defined on the sample space are given by the following tables: X: 0, 1 — $P(X)$: 0.2, 0.8 Y: 1, 2, 3 — $P(Y)$: 0.1, 0.4, 0.5 Find the joint probability of X and Y and verify that COV(X,Y)=0	*1*	*1*	**7**
		UNIT - 5			
9	a)	A machine runs on an average of 125 hours/year. A random sample of 49 machines has an annual average use of 126.9 hours with standard deviation 8.4 hours. Does this suggest to believe that machines are used on the average more than 125 hours annually at 0.05 level of significance?	*1*	*1*	**6**
	b)	The average weekly losses of man-hours due to strikes in an institute before and after a disciplinary program was implemented are as follows: Is there a reason to believe that the disciplinary program is effective at 1% level of significance? Before: 45, 73, 46, 124, 33, 57, 83, 34, 26, 17 After: 36, 60, 44, 119, 35, 51, 77, 29, 24, 11	*1*	*1*	**7**
	c)	A sample analysis of examination results of 500 students was made. It was found that 220 students had failed, 170 had secured third class, 90 had secured second class and 20 had secured first class. Do these figures support the general examination result which is in the ratio 4:3:2:1 for the respective categories? Use the level of significance $\alpha = 5\%$.	*1*	*1*	**7**
		OR			
10	a)	An auditor claims that he takes on an average 10.5 days to file income tax returns (I.T. returns). Can this claim be accepted if a random sample shows that he took 13, 19, 15, 10, 12, 11, 14, 18 days to file I.T. returns? Use 0.01 level of significance.	*1*	*1*	**6**
	b)	If a random sample data show that 42 men earn on the average $\bar{x}_1 = 744.85$ with S.D. $s_1 = 397.7$ while 32 women earn on the average $\bar{x}_2 = 516.78$ with S.D. $s_2 = 162.523$, test at 0.05 level of significance whether the average income for men and women is same or not.	*1*	*1*	**7**
	c)	Fit a Poisson distribution to the following data and test for the goodness of fit at $\alpha = 5\%$. x: 0, 1, 2, 3, 4 f: 419, 352, 154, 56, 19	*1*	*1*	**7**

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