U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

June 2025 Semester End Main Examinations

Programme: B.E. **Semester: IV**
Course Code/Branch: **Duration: 3 hrs.**
23MA4BSCPS (AS/ME /ECE/ETE/EIE)
22MA4BSCPS (AS/ME/EEE/ECE/ETE/EIE/MD/CV)
Course: Complex Analysis, Probability and Statistical Methods **Max Marks: 100**

Instructions:
1. Each unit has an internal choice; answer one complete question from each unit.
2. Missing data, if any, may be suitably assumed.
3. Use of statistical tables are permitted.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	CO	PO	Marks
1	a)	If $\varphi + i\psi$ represents the complex potential of an electrostatic field where $\psi = (x^2 - y^2) + \frac{x}{x^2+y^2}$, find the complex potential as a function of the complex variable z and hence determine φ.	1	1	6
	b)	Verify Cauchy's theorem for the function $f(z) = z^2$ over the boundary of square having vertices $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$.	1	1	7
	c)	State and prove Cauchy-Riemann equations in polar form and hence deduce that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$.	1	1	7
		OR			
2	a)	Discuss the conformal transformation $w = z^2$.	1	1	6
	b)	Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)} dz$ over the circle i) $\lvert z \rvert = 3$ ii) $\lvert z \rvert = \frac{3}{2}$.	1	1	7
	c)	If $f(z)$ is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\lvert f(z)\rvert^2 = 4\lvert f'(z)\rvert^2$.	1	1	7
		UNIT - 2			
3	a)	Express $f(x) = 4x^3 - 2x^2 - 3x + 8$ in terms of Legendre polynomial.	1	1	6
	b)	Obtain the series solution of Bessel's differential equation.	1	1	7
	c)	Prove that (i) $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ (ii) $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.	1	1	7

OR

4 a) Prove that $i)\ J_{-n}(x) = (-1)^n J_n(x)$ $\ ii)\ J_n(-x) = (-1)^n J_n(x)$ where n is a positive integer. — 1 | 1 | **6**

b) Obtain the series solution of Legendre's differential equation. — 1 | 1 | **7**

c) Derive the generating function for the Bessel function $J_n(x)$ in the form $e^{\frac{x}{2}(t-1/t)} = \sum_{n=-\infty}^{\infty} J_n(x)t^n$. — 1 | 1 | **7**

UNIT - 3

5 a) If P is the pull required to lift a load W by means of a pulley block, find a linear relation of the form $P = a + bW$ connecting P and W using the following data: — 1 | 1 | **6**

P	50	70	100	120
W	12	15	21	25

Estimate P when W is 150.

b) While calculating correlation co-efficient between two variables x and y from 25 pairs of observations, the following results were obtained: $\sum x = 125$, $\sum x^2 = 650$, $\sum y = 100$, $\sum y^2 = 460$ and $\sum xy = 508$. Later it was discovered at the time of checking that the pairs of values $\begin{matrix} x: & 8 & 6 \\ y: & 12 & 8 \end{matrix}$ were copied down as $\begin{matrix} x: & 6 & 8 \\ y: & 14 & 6 \end{matrix}$. Obtain the correct value of correlation coefficient. — 1 | 1 | **7**

c) Fit a least squares geometric curve $y = a x^b$ to the following data. — 1 | 1 | **7**

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

OR

6 a) Ten competitors in a contest are ranked by two judges as follows: — 1 | 1 | **6**

x:	1	6	5	10	3	2	4	9	7	8
y:	6	4	9	8	1	2	3	10	5	7

Compute the coefficient of rank correlation.

b) If θ is the angle between the two regression lines relating the variables x and y, then show that $\tan\theta = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\left(\frac{1-r^2}{r}\right)$. Explain its significance when $r = 0$ and $r = \pm 1$. — 1 | 1 | **7**

c) If the velocity V (km/hr) and resistance R (kg/ton) are related by $R = a + bV^2$, find a and b by the method of least squares using the following table. — 1 | 1 | **7**

V	10	20	30	40	50
R	8	10	15	21	30

<table>
<tr><td></td><td></td><td></td><td>UNIT - 4</td><td></td><td></td><td></td></tr>
<tr><td></td><td>7</td><td>a)</td><td>In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) no defective (ii) one defective and (iii) two defective blades respectively in a consignment of 10,000 packets.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>The joint distribution of two random variables X and Y are given by the following table.
<table>
<tr><td>X \ Y</td><td>- 4</td><td>2</td><td>7</td></tr>
<tr><td>1</td><td>$\frac{1}{8}$</td><td>$\frac{1}{4}$</td><td>$\frac{1}{8}$</td></tr>
<tr><td>5</td><td>$\frac{1}{4}$</td><td>$\frac{1}{8}$</td><td>$\frac{1}{8}$</td></tr>
</table>
Determine(i) The marginal distributions of X and Y
(ii) $E(X)$ and $E(Y)$ (iii) $E(XY)$ (iv) $Cov(X,Y)$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>8</td><td>a)</td><td>The mean height of 500 students is 151 cm and the standard deviation is 15 cm. Assuming that the heights are normally distributed, find how many are having heights between 120 and 155 cm.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Obtain an expression of the mean and variance for Poisson distribution.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>A coin is tossed three times. Let X be equal to 0 or 1 according as a head or a tail occurs on the first toss. Let Y be equal to the total number of heads which occur. Determine
i) the marginal distributions of X and Y
ii) the joint distribution of X and Y
iii)find the expected values of X, Y and XY.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 5</td><td></td><td></td><td></td></tr>
<tr><td></td><td>9</td><td>a)</td><td>The length of life X of certain computers is approximately normally distributed with mean 800 hours and standard deviation 40 hours. If a random sample of 30 computers has an average life of 788 hours, test the null hypothesis that μ=800 hours against the alternative that $\mu \neq 800$ hours at 5% level of significance.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>In a Mathematics examination 9 students of class A and 6 students of class B obtained the following marks. Test at 0.01 level of significance whether the performance in Mathematics is same or not for the two classes A and B. Assume that the samples are drawn from normal populations having same variance.
<table>
<tr><td>A:</td><td>44</td><td>71</td><td>63</td><td>59</td><td>68</td><td>46</td><td>69</td><td>54</td><td>48</td></tr>
<tr><td>B:</td><td>52</td><td>70</td><td>41</td><td>62</td><td>36</td><td>50</td><td>--</td><td>--</td><td>--</td></tr>
</table></td><td>1</td><td>1</td><td>7</td></tr>
</table>

		c)	Test for goodness of fit of a Poisson distribution at 5% level of significance to the following frequency distribution: *x*: 0, 1, 2, 3, 4 *f*: 122, 60, 45, 22, 12	*1*	*1*	**7**
			OR			
	10	a)	An ambulance service company claims that on an average it takes 20 minutes between a call for an ambulance and the patient's arrival at the hospital. If in 6 calls the time taken (between a call and arrival at hospital) are 27, 18, 26, 15, 20 and 32. Can the company's claim be accepted at 1% level of significance?	*1*	*1*	**6**
		b)	In a random sample of 100 tube lights produced by company A, the mean lifetime (mlt) of tube light is 1190 hours with standard deviation of 90 hours. Also in a random sample of 75 tube lights from company B the mean lifetime is 1230 hours with standard deviation of 120 hours. Is there a difference between the mean lifetimes of the two brands of tube lights at 0.05 level of significance?	*1*	*1*	**7**
		c)	A machine is supposed to mix peanuts, hazelnuts, cashews and almonds in the ratio 5:2:2:1. A tin containing 500 of these mixed nuts was found to have 269 peanuts, 112 hazelnuts, 74 cashews and 45 almonds. Can we conclude that the machine is mixing the nuts in the stated ratio?	*1*	*1*	**7**

x	0	1	2	3	4
f	122	60	45	22	12

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