| U.S.N. | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|

# B.M.S. College of Engineering, Bengaluru-560019

**Autonomous Institute Affiliated to VTU**

## September / October 2023 Supplementary Examinations

**Programme: B.E** **Semester: IV**
**Branch: AS/CV/EEE/ECE/EIE/ML/ETE** **Duration: 3 hrs.**
**Course Code: 19MA4BSEM4** **Max Marks: 100**
**Course: Engineering Mathematics-4** **Date: 14.09.2023**

**Instructions**: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.



### UNIT - I

1 a) If $\theta$ is the acute angle between the regression lines relating to the variables $x$ and $y$ then show that $\tan(\theta)=\left(\frac{1-r^2}{r}\right)\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$. 6

b) Using the method of least squares, fit a relation of the form $y=ab^x$ for the data: 7

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $y$ | 87 | 97 | 113 | 129 | 202 | 195 | 193 |

c) In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. Given that $A(0.5)=0.19$ and $A(1.4)=0.42$, where $A(z)$ is the area under the standard normal curve from 0 to $z$. 7

### UNIT - II

2 a) Prove that the matrix $P=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1/2 & 1/2 & 0\end{bmatrix}$ is a regular stochastic matrix. 6

b) The joint probability distribution of two random variables $X$ and $Y$ is given below. Find the marginal distribution of $X$ and $Y$ and $Cov(X,Y)$. 7

| X \ Y | - 3 | 2 | 4 |
|---|---|---|---|
| 1 | 0.1 | 0.2 | 0.2 |
| 3 | 0.3 | 0.1 | 0.1 |

c) Three boys A, B, C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C. C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball, find the probability that after three throws 7
(i) A has the ball (ii) B has the ball and (iii) C has the ball.

**Important Note:** Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

## UNIT - III

3 a) Derive the explicit formula for the solution of one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$. 6

b) Solve the initial boundary value problem $u_t = u_{xx}$, up to four-time levels when $u(0,t) = 0 = u(1,t)$, $t \geq 0$ $u(x,0) = \begin{cases} 2x, & 0 \leq x \leq 1/2 \\ 2(1-x), & 1/2 \leq x \leq 1 \end{cases}$ using Bendre-Schmidt method by taking $h = 1/4$. 7

c) The transverse displacement $u$ of a point at a distance $x$ from one end at any time $t$ of a vibrating string satisfies the equation $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(0,t) = 0$, $u(5,t) = 0, \text{t} \geq 0,$ $u(x,0) = x(5-x)$, $0 \leq x \leq 5$ and $u_t(x,0) = 0$. Solve this equation numerically up to two-time levels, with $h = 1$ and $k = 0.5$. 7

## UNIT - IV

4 a) Find the harmonic conjugate of $v = \left(r - \frac{1}{r}\right)\sin(\theta)$, $r \neq 0$. 6

b) Determine the analytic function $f(z)$ as a function of $z$, given $u - v = (x-y)(x^2 + 4xy + y^2)$. 7

c) Discuss the transformation w = $z^2$. 7

**OR**

5 a) Derive Cauchy-Riemann equations in cartesian form. Also show that the real part of an analytic function $f(z) = u + iv$ is harmonic. 6

b) If $f(z)$ is a regular function, then prove that 7

$$\left\{\frac{\partial}{\partial x}|f(z)|\right\}^2 + \left\{\frac{\partial}{\partial y}|f(z)|\right\}^2 = |f'(z)|^2.$$

c) Find the bilinear transformation which maps the points $\infty$, $i$, $0$ of the Z-plane onto the points -1, -*i*, 1 of the W-plane respectively. Also find the invariant points of the transformation. 7

## UNIT - V

6 a) State and prove Cauchy's integral formula. 6

b) Expand the function $f(z) = \frac{z}{(z-1)(z-3)}$ in power series in the following regions: (a) $|z| < 1$ and (b) $1 < |z| < 3$. 7

c) Apply Cauchy's residue theorem to evaluate $\oint_C \frac{z^2}{(z-1)^2(z+2)} dz$, where $C$ is the circle $|z| = \frac{5}{2}$. 7

**OR**

7 a) Evaluate $\int_{0}^{2+i} (\bar{z})^2 \, dz$ along the real axis to 2 and then vertically to $2+i$. 6

b) Evaluate $\int_{c} \frac{4-3z}{z(z-1)(z-2)} dz$ where $c: |z| = \frac{3}{2}$ using Cauchy's integral formula. 7

c) Find the poles and residues at each pole for $f(z) = \frac{z^2}{z^4 - 1}$ which lies inside $|z| = 2$. 7

******