U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

September / October 2024 Supplementary Examinations

Programme: B.E. **Semester: IV**
Branch: AS / CV / EEE / ECE / EIE / ML / TCE **Duration: 3 hrs.**
Course Code: 19MA4BSEM4 **Max Marks: 100**
Course: Engineering Mathematics-4

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	CO	PO	Marks
1	a)	If θ is the angle between the two regression lines, show that $\tan\theta = \frac{1-r^2}{r}\cdot\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$.	1	1	**6**
	b)	Obtain the lines of regression and hence find the coefficient of correlation for the data. x: 1, 3, 4, 2, 5 y: 8, 6, 10, 8, 12	1	1	**7**
	c)	In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and S.D of 60 hours. Estimate the number of bulbs likely to burn for: a) more than 2150 hours, b) less than 1950 hours where $P(0<z<1.83)=0.4664$ and $P(0<z<1.33)=0.0918$.	1	1	**7**
		UNIT – 2			
2	a)	Verify whether the matrix $A=\begin{bmatrix}0 & 0 & 1\\ 0.5 & 0.25 & 0.25\\ 0 & 1 & 0\end{bmatrix}$ is a regular stochastic matrix.	1	1	**6**
	b)	The joint distribution of two random variables X and Y are given below: X \ Y: -3, 2, 4 1: 0.1, 0.2, 0.2 2: 0.3, 0.1, 0.1 Find the marginal distributions of X, Y and $Cov(X, Y)$.	1	1	**7**
	c)	A student's study habits are as follows. If he studies one night, he is 60% sure not to study the next night. On the other hand, if he does not study one night, he is 80% sure to study the next night. In the long run, how often does he study.	1	1	**7**

			UNIT – 3			
	3	a)	Derive the Crank-Nicloson formula to solve the one-dimensional heat equation $u_t = c^2 u_{xx}$.	2	1	**6**
		b)	Solve $u_{tt} = u_{xx}$, $0 \le x \le 1$, subject to the initial conditions $u(x,0) = \sin \pi x$, $u_t(x,0) = 0$ and the boundary conditions $u(0,t) = 0$, $u(1,t) = 0$ using $h = 0.2$ and $k = 0.2$. Carry out computations for two time-levels.	2	1	**7**
		c)	Solve numerically the equation $u_t = u_{xx}$ subject to the conditions $u(0,t) = 0 = u(1,t)$; $t \ge 0$ and $u(x,0) = 2x - x^2$, $0 \le x \le 2$. Carry out computations for two time-levels taking $h = 0.5$, $k = \frac{1}{8}$.	2	1	**7**
			UNIT - 4			
	4	a)	Determine the analytic function $f(z) = u + iv$ whose real part is $u = x^3 - 3xy^2 + 3x^2 - 3y^2$.	3	1	**6**
		b)	If $f(z)$ is a analytic function of z, then prove that $\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right] \lvert f(z) \rvert^2 = 4 \lvert f'(z) \rvert^2$.	3	1	**7**
		c)	Discuss the transformation $w = z^2$.	3	1	**7**
			OR			
	5	a)	Show that polar form of Cauchy-Riemann equations are $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$, $\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$.	3	1	**6**
		b)	Verify that the function $u = y + e^x \cos(y)$ is harmonic and hence find its harmonic conjugate.	3	1	**7**
		c)	Find the bilinear transformation that maps $z = \infty, i, 0$ onto $w = -1, -i, 1$.	3	1	**7**
			UNIT - 5			
	6	a)	State and prove Cauchy's theorem.	3	1	**6**
		b)	Evaluate $\int_C \frac{e^z}{z + i\pi} dz$ over the following contours C: $(i) \lvert z \rvert = 2\pi$ $(ii) \lvert z \rvert = \pi/2$ by Cauchy's integral formula.	3	1	**7**
		c)	Expand the function $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent series valid for $(a)\ 0 < \lvert z+1 \rvert < 2$, $(b)\ \lvert z+1 \rvert > 2$.	3	1	**7**
			OR			
	7	a)	State and prove Cauchy's integral formula.	3	1	**6**
		b)	Verify Cauchy's theorem for the function $f(z) = z^2$ where C is the square having vertices $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$.	3	1	**7**
		c)	Apply Cauchy's residue theorem to evaluate the integral $\int_C \frac{z^2}{(z-1)(z+2)}$, where C is the circle $\lvert z \rvert = \frac{5}{2}$.	3	1	**7**