U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

January / February 2025 Semester End Main Examinations

Programme: B.E. **Semester: IV**
Branch: AS/CV/EEE/ECE/EIE/ML/TCE **Duration: 3 hrs.**
Course Code: 19MA4BSEM4 **Max Marks: 100**
Course: ENGINEERING MATHEMATICS - 4

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	CO	PO	Marks
1	a)	Apply the method of least square to fit a curve $y = a + bx$ for the following data. x: 1, 2, 3, 4, 5, 6 y: 3, 4, 5, 6, 7, 8	1	1	6
	b)	A book contains 100 misprints distributed randomly throughout its 100 pages. Assuming that the number of misprints follows Poisson distribution. What is probability that a page observed at random contains (i) no misprints (ii) at least two misprints.	1	1	7
	c)	In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and standard deviation of 60 hours. Estimate the number of bulbs likely to burn for i) More than 2150 hours (ii) Less than 1950 hours (iii) More than 1920 hours and less than 2160 hours (Assume P(1.5)=0.4332, P(1.83)=0.4664, P(2)=0.4772)	1	1	7
		OR			
2	a)	Apply the method of least square to fit a second-degree polynomial of the form $y = a + bx + cx^2$ for the following data. x: 0, 1, 2, 3, 4 y: 1, 1.8, 1.3, 2.5, 6.3	1	1	6
	b)	In a partially destroyed laboratory record, only the lines of regression of y on x and x on y are available as $4x - 5y + 33 = 0$ and $20x - 9y = 107$ respectively. Calculate $\bar{x}$, $\bar{y}$ and the co-efficient of correlation between x and y.	1	1	7
	c)	Fit a Poisson distribution for the following data: x: 0, 1, 2, 3, 4, 5 f: 47, 31, 16, 3, 2, 1	1	1	7

<table>
<tr><td></td><td></td><td></td><td>UNIT - 2</td><td></td><td></td><td></td></tr>
<tr><td></td><td>3</td><td>a)</td><td>Prove that the given matrix $P=\begin{bmatrix}0 & 2/3 & 1/3\\ 1/2 & 0 & 1/2\\ 1/2 & 1/2 & 0\end{bmatrix}$ is a regular stochastic matrix.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>The joint distribution of two random variables X and Y is as follows
<table>
<tr><td>X/Y</td><td>-3</td><td>2</td><td>4</td></tr>
<tr><td>1</td><td>0.1</td><td>0.2</td><td>0.2</td></tr>
<tr><td>3</td><td>0.3</td><td>0.1</td><td>0.1</td></tr>
</table>
(i) Find the marginal probability distributions of X and Y,
(ii) COV(X, Y).</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>In a certain city, the weather on a day is reported as sunny, cloudy or rainy. If a day is sunny, the probability that the next day is sunny is 70%, cloudy is 20% and rainy is 10%. If a day is cloudy, the probability that the next day is sunny is 30%, cloudy is 20% and rainy is 50%. If a day is rainy, the probability that the next day is sunny is 30%, cloudy is 30% and rainy is 40%. (i) If the Sunday is sunny, find the probability that the Wednesday is rainy (ii) Find the steady state distribution.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>4</td><td>a)</td><td>If the joint probability distribution of X and Y is given by
$f(x,y)=C(x^2+y^2)$ for $x=-1,\ 0,\ 1,\ 3;\quad y=-1,\ 2,\ 3.$
Determine (i) the value of the constant C, (ii) $P(X\leq 1, Y>2)$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Gambler's luck follows a pattern. If he wins a game, the probability of winning the next game is 0.6. However, if he loses a game, the probability of losing the next game is 0.7. There is an even chance of gambler winning the first game.
(i) What is the probability of he winning the second game?
(ii) What is the probability of he winning the third game?</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Find the unique fixed probability vector for the matrix
$P=\begin{bmatrix}1/2 & 1/4 & 1/4\\ 1/2 & 0 & 1/2\\ 0 & 1 & 0\end{bmatrix}.$</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 3</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Derive the numerical solution of one-dimensional wave equation $u_{tt}=c^2u_{xx}$.</td><td>2</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Apply Schmidt method to solve the equation $u_t=u_{xx}$, subject to the conditions $u(0,t)=0=u(1,t),\ t\geq 0$ and $u(x,0)=\sin\pi x,\ 0\leq x\leq 1$. Carry out the computations for two-time levels taking $h=\frac{1}{3}, k=\frac{1}{36}$.</td><td>2</td><td>1</td><td>7</td></tr>
</table>

		c)	Evaluate the pivotal values of the equation $u_{tt}=16u_{xx}$ taking $h=1$, $k=\frac{1}{4}$ upto $t=\frac{1}{2}$. The boundary conditions are $u(0,t)=0=u(5,t)$, $t\geq 0$; $u_t(x,0)=0$ and $u(x,0)=x^2(5-x)$.	*2*	*1*	**7**
			OR			
6	a)		Derive Crank-Nicolson formula to numerically solve the one-dimensional heat equation $u_t=c^2u_{xx}$.	*2*	*1*	**6**
		b)	Apply Bendre-Schmidt method to solve the heat equation $u_t=4u_{xx}$ subjected to conditions $u(0,t)=0=u(8,t)$, $u(x,0)=4x-\frac{x^2}{2}$, $0\leq x\leq 8$ taking $h=1$ and $k=\frac{1}{8}$. Carry out the computation up to $t=\frac{2}{8}$.	*2*	*1*	**7**
		c)	Solve the wave equation $u_{tt}=u_{xx}$, given that $u(0,t)=0=u(1,t)$, $u_t(x,0)=0$ and $u(x,0)=\sin\pi x$ for $0\leq x\leq 1$, by taking h=0.25 and k=0.2. Compute up to two-time levels.	*2*	*1*	**7**
			UNIT - 4			
7	a)		Derive Cauchy-Riemann equation in Cartesian form.	*3*	*1*	**6**
		b)	Construct the analytic function $f(z)$ whose real part is $u(x,y)=x^2-y^2+\frac{x}{x^2+y^2}$ using Milne-Thompson method.	*3*	*1*	**7**
		c)	Find the bilinear transformation which maps the points $0,-i,-1$ onto the points $i,1,0$ respectively.	*3*	*1*	**7**
			OR			
8	a)		If $f(z)$ is a regular function of z, then prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\lvert f(z)\rvert^2=4\lvert f'(z)\rvert^2$.	*3*	*1*	**6**
		b)	Find the harmonic conjugate of $u(r,\theta)=-r^3\sin 3\theta$ by constructing its analytic function $f(z)=u(r,\theta)+i\,v(r,\theta)$ using Milne-Thompson method.	*3*	*1*	**7**
		c)	Discuss the conformal transformation $w=z^2$.	*3*	*1*	**7**
			UNIT – 5			
9	a)		State and prove Cauchy's Integral formula.	*3*	*1*	**6**
		b)	Obtain the Laurent's series expansion $f(z)=\frac{z+3}{z(z+1)(z-2)}$ in the region $1<\lvert z\rvert<2$.	*3*	*1*	**7**
		c)	Verify Cauchy's theorem for the integral of $\frac{1}{z}$ taken over the boundary of the triangle having vertices $(1,2)$, $(1,4)$ and $(3,2)$.	*3*	*1*	**7**
			OR			

	10	a)	State and prove Cauchy's theorem.	*3*	*1*	**6**
		b)	Evaluate $\int_C \frac{dz}{z^2-4}$ where C is the circle $\lvert z \rvert = 3$	*3*	*1*	**7**
		c)	Apply Cauchy residue theorem to evaluate $\int_C \frac{\sin(\pi z^2)+\cos(\pi z^2)}{(z-1)^2(z-2)}dz$ where C is the circle $\lvert z \rvert = 3$.	*3*	*1*	**7**

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