U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

October 2023 Semester End Main Examinations

Programme: B.E. **Semester: IV**
Branch: AS/CV/EEE/ECE/EIE/ETE/MD **Duration: 3 hrs.**
Course Code: 19MA4BSEM4 **Max Marks: 100**
Course: ENGINEERING MATHEMATICS- 4

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - I	*CO*	*PO*	Marks
1	a)	2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses contains (i) No defective fuses. (ii) 3 or more defective fuses.	CO1	PO1	**6**
	b)	Apply the method of least squares to fit a relation of the form $y = ab^x$ for the data: $\begin{array}{\|c\|c\|c\|c\|c\|c\|c\|} \hline x & 2 & 4 & 6 & 8 & 10 & 12 \\ \hline y & 1.8 & 1.5 & 1.4 & 1.1 & 1.1 & 0.9 \\ \hline \end{array}$	CO1	PO1	**7**
	c)	If θ is the acute angle between the regression lines relating to the variables x and y then show that $tan\,\theta = \left(\frac{1-r^2}{r}\right)\frac{\sigma_x \sigma_y}{\sigma_x^2+\sigma_y^2}$. Indicate the significance when $r = \pm 1$ and $r = 0$.	CO1	PO1	**7**
		UNIT - II			
2	a)	Find the joint distribution of X and Y, which are independent random variables with the following respective distributions. Also find $Cov(X,Y)$. $\begin{array}{\|c\|c\|c\|} \hline X & 1 & 2 \\ \hline f(X) & 0.7 & 0.3 \\ \hline \end{array}$ $\begin{array}{\|c\|c\|c\|c\|} \hline Y & -2 & 5 & 8 \\ \hline g(Y) & 0.3 & 0.5 & 0.2 \\ \hline \end{array}$	CO1	PO1	**6**
	b)	Prove that the Markov chain whose transition probability matrix $P = \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 1/4 & 1/4 \\ 0 & 1 & 0 \end{bmatrix}$ is irreducible. Find the corresponding fixed probability vector.	CO1	PO1	**7**
	c)	In a certain city, the weather on a day is reported as sunny, cloudy or rainy. If a day is sunny, the probability that the next day is sunny is 70%, cloudy 20% and rainy is 10%. If a day is	CO1	PO1	**7**

			cloudy, the probability that the next day is sunny is 30%, cloudy is 20% and rainy is 50%. If a day is rainy, the probability that the next day is sunny is 30%, cloudy is 30% and rainy is 40%. Find the transition matrix for the change of weather. If a Sunday is sunny, find the probability that the Wednesday is cloudy.			
			UNIT - III			
	3	a)	Derive Crank Nicolson implicit formula for the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$.	CO2	PO1	**6**
		b)	The transverse displacement u of a point at a distance x from one end at any time t of a vibrating string satisfies the equation $\frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(0,t) = 0$, $u(4,t) = 0, t \geq 0$, $u(x,0) = x^2(5-x)$ $0 \leq x \leq 4$ and $\frac{\partial u(x,0)}{\partial t} = 0$. Solve this equation numerically up to two-time levels, with $h = 1$ and $k = 0.25$.	CO2	PO1	**7**
		c)	Solve $u_t = u_{xx}$ by Bendre-Schmidt method subject to conditions $u(0,t) = 0,\ u(1,t) = 0$ and $u(x,0) = \begin{cases} 2x, & 0 \leq x \leq 0.5 \\ 2(1-x), & 0.5 \leq x \leq 1 \end{cases}$ taking $h = 1/4$ and $k = 1/32$ up to 3 time levels.	CO2	PO1	**7**
			UNIT - IV			
	4	a)	Show that $v = -r^3 \sin 3\theta$ is harmonic and hence find the analytic function $f(z)$.	CO3	PO1	**6**
		b)	If $f(z)$ is a regular function, then prove that $\left\{\frac{\partial}{\partial x}\lvert f(z)\rvert\right\}^2 + \left\{\frac{\partial}{\partial y}\lvert f(z)\rvert\right\}^2 = \lvert f'(z)\rvert^2$.	CO3	PO1	**7**
		c)	Discuss the transformation w = z^2.	CO3	PO1	**7**
			OR			
	5	a)	If $f(z)$ is an analytic function with constant modulus, then prove that $f(z)$ is constant.	CO3	PO1	**6**
		b)	Determine the analytic function $f(z)$ as function of z, given $u + v = x^3 - y^3 + 3xy(x-y)$.	CO3	PO1	**7**
		c)	Find the bilinear transformation which maps the points $\infty, i, 0$ of the Z-plane onto the points $-1, -i, 1$ of the W-plane respectively. Also, find the invariant points of the transformation.	CO3	PO1	**7**
			UNIT - V			
	6	a)	State and prove Cauchy's integral formula.	CO3	PO1	**6**

		b)	Expand the function $f(z)=\frac{z}{(z-1)(z-3)}$ as a Laurent's series in the following regions: (a) $\lvert z\rvert<1$ (b) $1<\lvert z\rvert<3$.	CO3	PO1	**7**
		c)	Apply Cauchy's residue theorem, evaluate $\oint_C \frac{z^2}{(z-1)^2(z+2)}dz$, where C is the circle $\lvert z\rvert=5/2$.	CO3	PO1	**7**
			OR			
	7	a)	Evaluate $\oint_C \frac{e^{2z}}{(z-1)(z-2)}\,dz$ where C is $\lvert z\rvert=3$ using Cauchy's integral formula.	CO3	PO1	**6**
		b)	Evaluate $\oint_C \lvert z\rvert^2\,dz$ where C is a square with vertices $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$.	CO3	PO1	**7**
		c)	Find the poles and residues at each pole for $f(z)=\frac{z^2}{z^4-1}$ which lies inside $\lvert z\rvert=2$.	CO3	PO1	**7**

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