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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

June 2025 Semester End Main Examinations

Programme: B.E. **Semester: IV**
Branch: Common to all Branches **Duration: 3 hrs.**
Course Code: 19MA4BSEM4 **Max Marks: 100**
Course: Engineering Mathematics – 4

Instructions: 1. Each unit has an internal choice; answer one complete question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	*CO*	*PO*	Marks
1	a)	The results of measurement of electric resistance R of a copper bar at various temperatures $t^0 C$ are listed below: t: 19, 25, 30, 36, 40, 45, 50 R: 76, 77, 79, 80, 82, 83, 85 Find a relation $R = a + bt$ where a & b are constants.	*1*	*1*	**6**
	b)	In a partially destroyed laboratory record, only the lines of regression of y on x and x on y are available as $4x-5y+33=0$ and $20x-9y=107$ respectively. Calculate $\bar{x}, \bar{y}$ and the correlation coefficient between x and y.	*1*	*1*	**7**
	c)	Derive an expression for the mean and variance of the Poisson distribution.	*1*	*1*	**7**
		OR			
2	a)	Fit a least square of the form $y = a + bx^2$ for the following data. x: 1, 2.5, 3.5, 4 y: 3.8, 15, 26, 33	*1*	*1*	**6**
	b)	Find the co-efficient of correlation between industrial production (x: in crore tons) and export (y: in crore tons) using the following data: x: 55, 56, 58, 59, 60, 60, 62 y: 35, 38, 38, 39, 44, 43, 45	*1*	*1*	**7**
	c)	In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and standard deviation of 60 hours. Estimate the number of bulbs likely to burn for i) more than 2150 hours ii) less than 1950 hours iii) more than 1920 hours but less than 2160 hours. Given: $A(1.33) = 0.4082$, $A(2) = 0.4772$ and $A(1.83) = 0.4664$.	*1*	*1*	**7**

			UNIT - 2			
	3	a)	A housewife buys 3 kinds of cereals A, B and C. She never buys the same cereal in successive weeks. If she buys cereal A, the next week she buys cereal B. However, if she buys cereal B or C , the next week she is three times as likely to buy cereal A as the other cereal. In the long run how often, she buys each of the three cereals?	*1*	*1*	**6**
		b)	Find i) marginal distributions $f(x)$ and $g(y)$ ii) $E(x)$ and $E(y)$ iii) $\text{Cov}(x,y)$ for the following distribution. $x \backslash y$: -4, 2, 7 1 : $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{8}$ 5 : $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{8}$	*1*	*1*	**7**
		c)	Prove that the given stochastic matrix $P = \begin{pmatrix} 0 & 3/4 & 1/4 \\ 1/2 & 1/2 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ is regular and hence find its fixed probability vector.	*1*	*1*	**7**
			OR			
	4	a)	The distributions of two stochastically independent random variables X and Y defined on the same sample space are given by the following table: X : 0, 1 ; $P(X)$: 0.2, 0.8 Y : 1, 2, 3 ; $P(Y)$: 0.1, 0.4, 0.5 Find the joint distribution of X and Y. Also, evaluate $\text{Cov}(X,Y)$.	*1*	*1*	**6**
		b)	A coin is tossed three times. Let X denote 0 or 1 depending whether a tail or head occurs on the first toss. Let Y denote the total number of tails which occur. Determine the i) joint distribution of X and Y and ii) the correlation coefficient of X and Y.	*1*	*1*	**7**
		c)	A salesman's territory consists of three cities A, B and C. He never sells in the same city on successive days. If he sells in the city A, then the next day he sells in the city B. However, if he sells in either B or C then he is equally likely to sell in city A or city B or city C. In the long run, how often does he sell in each of the cities?	*1*	*1*	**7**
			UNIT - 3			
	5	a)	Derive the numerical solution of one-dimensional wave equation $u_{tt} = c^2 u_{xx}$.	*2*	*1*	**6**
		b)	Solve $u_{tt} = 4u_{xx}$ under conditions $u(0,t)=0$, $u(5,t)=0$, $\frac{\partial u}{\partial t}=0$ and $u = x(5-x)$, $0 \le t \le 5$, taking $h=1$ and $k=1/2$. Carryout the computations up to two-time levels.	*2*	*1*	**7**

	c)	Find the numerical solution of one-dimensional heat equation $u_{xx} = 2u_t$ when $u(0,t) = 0 = u(4,t)$ and $u(x,0) = x(4-x)$ by taking $h = 1$. Find the values of u upto $t = 5$, given $k = 1$.	*2*	*1*	**7**
		OR			
6	a)	Derive Bendre-Schmidt formula for the one-dimensional heat equation $u_t = c^2 u_{xx}$.	*2*	*1*	**6**
	b)	Solve numerically the equation $u_t = u_{xx}$ subject to the conditions $u(0,t) = 0 = u(1,t), t \geq 0$ and $u(x,0) = \sin \pi x, 0 \leq x \leq 1$. Using Crank-Nicolson method carry out computations for two levels taking $h = \frac{1}{3}$ and $k = \frac{1}{36}$.	*2*	*1*	**7**
	c)	Solve numerically the equation $u_{tt} = 4u_{xx}$ subject to the conditions $u(0,t) = u(5,t) = 0, t \geq 0$, $u(x,0) = x(5-x)$ and $u_t(x,0) = 0$. Carry out computations for two levels taking $h = 1$ and $k = 0.5$.	*2*	*1*	**7**
		UNIT - 4			
7	a)	Derive Cauchy-Riemann equations in cartesian form.	*3*	*1*	**6**
	b)	Discuss the conformal transformation $w = z + \frac{k^2}{z}$.	*3*	*1*	**7**
	c)	Find the orthogonal trajectories of the family of curves $r^2 \cos 2\theta = c_1$.	*3*	*1*	**7**
		OR			
8	a)	If $f(z)$ is a holomorphic function of z, show that $\left\{\frac{\partial}{\partial x}\lvert f(z)\rvert\right\}^2 + \left\{\frac{\partial}{\partial y}\lvert f(z)\rvert\right\}^2 = \lvert f'(z)\rvert^2$.	*3*	*1*	**6**
	b)	If $\varphi + i\psi$ represents the complex potential of an electrostatic field where $\psi = (x^2 - y^2) + \frac{x}{x^2 + y^2}$, find the complex potential as a function of the complex variable z and hence determine φ.	*3*	*1*	**7**
	c)	Find the bilinear transformation that transforms the points $z = 1, i, -1$ on to the points $w = i, 0, -i$.	*3*	*1*	**7**
		UNIT - 5			
9	a)	State and prove Cauchy's theorem.	*3*	*1*	**6**
	b)	Obtain the Laurent's series expansion of $f(z) = \frac{7z-2}{(z+1)z(z-2)}$ in the region (i) $\lvert z\rvert < 1$ (ii) $\lvert z\rvert < 2$.	*3*	*1*	**7**
	c)	Evaluate $\int_C \frac{z^2 - z + 1}{z - 1}$ where C is the circle $(i)\ \lvert z\rvert = 1$ $(ii)\ \lvert z\rvert = \frac{1}{2}$.	*3*	*1*	**7**
		OR			

	10	a)	State and prove Cauchy's integral formula.	*3*	*1*	**6**
		b)	Using Residue theorem, evaluate $\int_C \frac{dz}{z^3(z-1)^2}$ where C is the circle $\lvert z \rvert = 2$.	*3*	*1*	**7**
		c)	Verify the Cauchy's theorem for the function $f(z) = z^2$ over the boundary of square having vertices (0,0), (1,0), (1,1) and (0,1).	*3*	*1*	**7**

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