U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

September / October 2024 Supplementary Examinations

Programme: B.E. **Semester: IV**
Branch: Mechanical Engineering **Duration: 3 hrs.**
Course Code: 19MA4BSHEM **Max Marks: 100**
Course: Higher Engineering Mathematics

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

<table>
<tr><th></th><th></th><th>UNIT - I</th><th>CO</th><th>PO</th><th>Marks</th></tr>
<tr><td>1</td><td>a)</td><td>Find the coefficient of correlation and the lines of regression for the following data:
<table>
<tr><td>x</td><td>36</td><td>23</td><td>27</td><td>28</td><td>28</td><td>29</td><td>30</td><td>31</td><td>33</td><td>35</td></tr>
<tr><td>y</td><td>29</td><td>18</td><td>20</td><td>22</td><td>27</td><td>21</td><td>29</td><td>27</td><td>29</td><td>28</td></tr>
</table></td><td>CO1</td><td>PO1</td><td>6</td></tr>
<tr><td></td><td>b)</td><td>Suppose it has been observed that, on an average, 180 cars per hour pass a specified point on a particular road in the morning rush hour. Due to impending roadworks it is estimated that congestion will occur closer to the city center if more than 5 cars pass the point in any one minute. What is the probability of congestion occurring?</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td>c)</td><td>The average number of acres burned by forest and range fires in a large New Mexico county is 4,300 acres per year, with a standard deviation of 750 acres. The distribution of the number of acres burned is normal. What is the probability that between 2,500 and 4,200 acres will be burned in any given year?</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td>2</td><td>a)</td><td>Fit a second degree polynomial to the following data:
<table>
<tr><td>x</td><td>1.0</td><td>1.5</td><td>2.0</td><td>2.5</td><td>3.0</td><td>3.5</td><td>4.0</td></tr>
<tr><td>y</td><td>1.1</td><td>1.3</td><td>1.6</td><td>2.0</td><td>2.7</td><td>3.4</td><td>4.1</td></tr>
</table></td><td>CO1</td><td>PO1</td><td>6</td></tr>
<tr><td></td><td>b)</td><td>From 25 pairs of observations for x and y the following data is obtained, $n = 25, \sum x = 125, \sum y = 100, \sum x^2 = 650,\ \sum y^2 = 460,$ $\sum xy = 508$. It was later found that two pairs of values
<table>
<tr><td>x</td><td>8</td><td>6</td></tr>
<tr><td>y</td><td>12</td><td>8</td></tr>
</table>
were copied down as
<table>
<tr><td>x</td><td>6</td><td>8</td></tr>
<tr><td>y</td><td>12</td><td>6</td></tr>
</table>
. Calculate the correct value of the coefficient of correlation.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
</table>

<table>
<tr><td></td><td></td><td>c)</td><td>In a normal distribution 31% of the items are under 45 and 8% of the items are over 64. Find the mean and standard deviation.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - II</td><td></td><td></td><td></td></tr>
<tr><td></td><td>3</td><td>a)</td><td>The joint probability distribution table for two random variables X and Y is as follows.
<table>
<tr><td>X \ Y</td><td>-4</td><td>2</td><td>7</td></tr>
<tr><td>1</td><td>1/8</td><td>1/4</td><td>1/8</td></tr>
<tr><td>5</td><td>1/4</td><td>1/8</td><td>1/8</td></tr>
</table>
Are X and Y independent? Find COV (X,Y).</td><td>CO1</td><td>PO1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find the fixed probability vector of the regular stochastic matrix $\begin{bmatrix}0.5 & 0.5 & 0\\ 0.5 & 0 & 0.5\\ 0 & 0.5 & 0.5\end{bmatrix}$.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Every year, a man trades his car for a new car. If he has Maruti, he trades it for Santro, however if he has Santro he is just as likely to trade it for a new Santro as to trade it for a Maruti or an Ambassador. In 2000 he bought his first car, which was a Santro
(i) find the probability that he has
(a) 2002 Santro (b) 2002 Maruti
(c) 2003 Ambassador (d) 2003 Santro
(ii) In the long run how often will he have a Santro.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - III</td><td></td><td></td><td></td></tr>
<tr><td></td><td>4</td><td>a)</td><td>Derive the Bendre-Schmidt formula for the solution of one-dimensional heat equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$.</td><td>CO2</td><td>PO1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Solve $u_{xx}=u_t$ subject to the condition $u(0,t)=0,\ u(1,t)=0$, $u(x,0)=sin\pi x$ for $0\le t\le 0.1$ by taking $h = 0.2$ and $k=0.02$.</td><td>CO2</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Solve $25\ u_{xx}=u_{tt}$ at the pivotal points given $u(0,t)=0=u(5,t)$, $u_t(x,0)=0$ and $u(x,0)=\begin{cases}20\ x & 0\le x\le 1\\ 5(5-x) & 1\le x\le 5\end{cases}$ by taking $h=1$ and $k=0.2$, compute $u(x,t)$ for $0\le x\le 1$, up to two time levels.</td><td>CO2</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - IV</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Derive C-R equations in Cartesian form.</td><td>CO3</td><td>PO1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Discuss the conformal transformation of the function $w=z+\frac{a^2}{z}$, $z\neq 0$.</td><td>CO3</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Find the bilinear transformation which maps the points $z=1,i,-1$ into the points $w=0,\ 1,\ \infty$.</td><td>CO3</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
</table>

	6	a)	Find an analytic function $f(z)=u+iv$ whose imaginary part is $v(r,\theta)=\left(r-\frac{1}{r}\right)\sin\theta$.	CO3	PO1	**6**
		b)	If $f(z)$ is analytic, show that $\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right] \|f(z)\|^2 = 4\left\|f^{/}(z)\right\|^2$.	CO3	PO1	**7**
		c)	Find the bilinear transformation which maps the points $z=\infty, i, 0$ into the points $w=-1, -i, 1$. What are the invariant points of the transformation.	CO3	PO1	**7**
			UNIT - V			
	7	a)	State and prove the Cauchy's theorem.	CO3	PO1	**6**
		b)	Obtain the Laurent series expansion of $\frac{e^z}{(z-1)(z-3)}$ in the following regions (i) $1 < \|z\| < 3$ (ii) $\|z-1\| < 2$	CO3	PO1	**7**
		c)	Using Cauchy's residue theorem evaluate $\int_c \frac{\sin\pi z^2+\cos\pi z^2}{(z-1)^2(z-2)}dz$ where C is the circle $\|z\| = 3$	CO3	PO1	**7**

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