U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

January / February 2025 Semester End Main Examinations

Programme: B.E. | **Semester: IV**
Branch: Mechanical Engineering | **Duration: 3 hrs.**
Course Code: 19MA4BSHEM | **Max Marks: 100**
Course: Higher Engineering Mathematics

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

<table>
<tr><th></th><th></th><th>UNIT - 1</th><th>CO</th><th>PO</th><th>Marks</th></tr>
<tr><td>1</td><td>a)</td><td>Fit a second degree parabola to the following data and estimate y at $x=8$.
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>y</td><td>1</td><td>1.8</td><td>1.3</td><td>2.5</td><td>2.3</td></tr>
</table></td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td>b)</td><td>Predict the mean relation dose of radiation at an altitude of 3000 feet by fitting a geometrical curve $y=ab^x$ for the following data.
<table>
<tr><td>Altitude(x)</td><td>50</td><td>450</td><td>780</td><td>1200</td><td>4400</td><td>4800</td><td>5300</td></tr>
<tr><td>Dose of radiation(y)</td><td>28</td><td>30</td><td>32</td><td>36</td><td>51</td><td>58</td><td>69</td></tr>
</table></td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td>c)</td><td>In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td>2</td><td>a)</td><td>If θ is an angle between the two regression lines then show that $\tan\theta=\frac{1-r^2}{r}\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$. Explain the significance when $r=0$ and $r=\pm1$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td>b)</td><td>Find the correlation coefficient between x and y from the given data:
<table>
<tr><td>x</td><td>78</td><td>89</td><td>97</td><td>69</td><td>59</td><td>79</td><td>68</td><td>57</td></tr>
<tr><td>y</td><td>125</td><td>137</td><td>156</td><td>112</td><td>107</td><td>138</td><td>123</td><td>108</td></tr>
</table></td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td>c)</td><td>In a certain factory turning out razor blades there is a small probability of $\frac{1}{500}$ for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) no defective (ii) at least one (iii) at most two defective blades in a consignment of 10,000 packets.</td><td>1</td><td>1</td><td>7</td></tr>
</table>

<table>
<tr><td></td><td></td><td></td><td>UNIT-2</td><td></td><td></td><td></td></tr>
<tr><td></td><td>3</td><td>a)</td><td>Find the unique fixed probability vector of $p = \begin{bmatrix} 0 & 3/4 & 1/4 \\ 1/2 & 1/2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>The joint probability distribution of two random variables X and Y are given as:
<table>
<tr><td>X \ Y</td><td>-2</td><td>-1</td><td>4</td><td>5</td></tr>
<tr><td>1</td><td>0.1</td><td>0.2</td><td>0</td><td>0.3</td></tr>
<tr><td>2</td><td>0.2</td><td>0.1</td><td>0.1</td><td>0</td></tr>
</table>
Compute (i) $E(X)$ and $E(Y)$ (ii) $E(XY)$ (iii) $COV(X,Y)$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>A students' study habits are as follows: If he studies one night he is 70% sure not to study next night. On the other hand, if he does not study one night, he is 60% sure not to study next night as well. Supposing that he studies on Monday night, find the probability that he does not study on Friday night. In the long run, how often does he study?</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>4</td><td>a)</td><td>Verify that the matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 0.5 & 0.25 & 0.25 \\ 0 & 1 & 0 \end{bmatrix}$ is a regular stochastic matrix.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>If X and Y are independent random variables with the following respective distribution. Find the joint distribution of X and Y. Also verify that
COV (X, Y) = 0.
<table>
<tr><td>x_i</td><td>1/8</td><td>1/4</td></tr>
<tr><td>$f(x_i)$</td><td>1/4</td><td>1/8</td></tr>
</table>
<table>
<tr><td>y_j</td><td>1</td><td>1/8</td><td>1/4</td></tr>
<tr><td>$g(y_j)$</td><td>5</td><td>1/4</td><td>1/8</td></tr>
</table></td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Every year, a man trades his car for a new car. If he has Maruti, he trades it for Santro. However if he has Santro, he is just as likely to trade it for a new Santro as to trade it for a Maruti or an Ambassador. In 2000 he bought his first car which was a Santro. Find the probability that he has
(i) 2002 Santro (ii) 2002 Maruti
(iii) 2003 Ambassador (iv) 2003 Santro</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 3</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Derive Crank-Nicolson two level implicit formula for the solution of one dimensional heat equation $u_t = c^2 u_{xx}$.</td><td>2</td><td>2</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Solve $u_{xx} = u_t$ subject to the condition $u(0,t) = 0$, $u(1,t) = 0$, $u(x,0) = sin\pi x$ for $0 \le t \le 0.1$ by taking $h = 0.2$ and $k = 0.02$.</td><td>2</td><td>2</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Find the Numerical solution $u(x,t)$ of wave equation $u_{tt} = 4u_{xx}$ at $t = 2$ under the conditions $u(0,t) = u(5,t) = 0$, $t \ge 0$, $u_t(x,0) = 0$ and $u(x,0) = x(5-x)$, $0 \le t \le 5$ by taking $h = 1$ and $k = 0.5$.</td><td>2</td><td>2</td><td>7</td></tr>
</table>

			OR			
	6	a)	Derive Schmidt finite difference formula for the solution of one dimensional heat equation $u_t = c^2 u_{xx}$.	*2*	*2*	**6**
		b)	Solve the initial boundary value problem $u_t = u_{xx}$, at $t = 0.002$ under the conditions $u(0,t) = 0 = u(1,t)$ and $u(0,t) = f(x)$, $0 \le x \le 1$ using Schmidt method by taking $h = 0.1, k = 0.001$, where $f(x) = \begin{cases} 2x, & 0 \le x \le 1/2 \\ 2(1-x) & 1/2 \le x \le 1 \end{cases}$.	*2*	*2*	**7**
		c)	Find the solution of the initial boundary value problem $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, $0 \le x \le 1$ subject to the initial condition $u(x,0) = \sin \pi x$, $u_t(x,0) = 0$ and the boundary condition $u(0,t) = u(1,t) = 0$, $t > 0$ by taking step size $h = k = \frac{1}{3}$ up to two time levels.	*2*	*2*	**7**
			UNIT - 4			
	7	a)	If $f(z)$ is an analytic function, then show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\lvert f(z)\rvert^2 = 4\lvert f'(z)\rvert^2$.	*3*	*3*	**6**
		b)	Discuss the conformal transformation of the function $w = z + \frac{a^2}{z}, z \neq 0$.	*3*	*3*	**7**
		c)	Show that the function $u = x^3 - 3xy^2 - 3x^2 + 3y^2 + 1$ is harmonic and find the harmonic conjugate. Also find the corresponding analytic function.	*3*	*3*	**7**
			OR			
	8	a)	Derive Cauchy Riemann equations in polar form.	*3*	*3*	**6**
		b)	Construct the analytic function $f(z) = u + iv$, given $u = \log\sqrt{x^2 + y^2}$ and hence find its imaginary part.	*3*	*3*	**7**
		c)	Show that $u = \left(r + \frac{1}{r}\right) cos\theta$ is harmonic and find its harmonic conjugate.	*3*	*3*	**7**
			UNIT - 5			
	9	a)	State and prove the Cauchy's theorem.	*3*	*3*	**6**
		b)	Using Cauchy's residue theorem evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z}{(z-1)^2 (z-2)} dz$ where C is the circle $\lvert z\rvert = 3$.	*3*	*3*	**7**
		c)	Find the Laurent's expansion of $f(z) = \frac{7z-2}{(z+1)z(z-2)}$ in the region $1 < z+1 < 3$.	*3*	*3*	**7**
			OR			
	10	a)	State and prove the Cauchy's integral formula.	*3*	*3*	**6**
		b)	Evaluate $\int_C \frac{z}{(z-1)^2 (z-3)} dz$ where C is $\lvert z\rvert = 2$ using Cauchy's Residue Theorem.	*3*	*3*	**7**

		c)	Obtain the Laurent series expansion of $f(z)=\dfrac{z+3}{z(z-1)(z+2)}$ in the region $1<\lvert z\rvert<2$.	3	3	**7**

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