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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

September / October 2023 Supplementary Examinations

Programme: B.E **Semester: IV**
Branch: Mechanical Engineering **Duration: 3 hrs.**
Course Code: 19MA4BSHEM **Max Marks: 100**
Course: Higher Engineering Mathematics **Date: 14.09.2023**

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

UNIT - I

1 a) The two regression equations of the variables x and y are $x = 19.13 - 0.87y$ and $y = 11.64 - 0.50x$. Find (i) mean of x (ii) mean of y (iii) the correlation co-efficient between x and y. **6**

b) Fit a curve of the form $y = ab^x$, in the least square sense, to the following data: **7**

x	0	2	4	5	7	10
y	100	120	256	390	710	1600

c) The number of accidents per day (x) as recorded in a textile industry over a period of 400 days is given. Fit a Poisson distribution for the data and calculate the theoretical frequencies. **7**

x	0	1	2	3	4	5
f	173	168	37	18	3	1

OR

2 a) If θ is the angle between the two regression lines, show that $\tan\theta = \left(\frac{1-r^2}{r}\right)\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$. Explain the significance when $r = 0$ and $r = \pm 1$. **6**

b) While calculating correlation co-efficient between two variables x and y from 25 pairs of observations, the following results were obtained: $n = 25$, $\sum x = 125$, $\sum x^2 = 650$, $\sum y = 100$, $\sum y^2 = 460$, $\sum xy = 508$. Later it was discovered at the time of checking that the pairs of values $\begin{matrix} x: & 8 & 6 \\ y: & 12 & 8 \end{matrix}$ were copied down as $\begin{matrix} x: & 6 & 8 \\ y: & 14 & 6 \end{matrix}$. Obtain the correct value of correlation co-efficient. **7**

c) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution, given that $A(0.5) = 0.19$ and $A(1.4) = 0.42$, where $A(z)$ is the area under the standard normal curve from 0 to $z > 0$. **7**

UNIT - II

3 a) A student's study habits are as follows. If he studies one night, he is 70% sure not to study the next night. On the other hand, if he does not study one night, he is 60% sure not to study the next night. In the long run how often does he study? **6**

b) The joint probability distribution of two random variables X and Y is given below: **7**

X \ Y	- 4	2	7
1	$1/8$	$1/4$	$1/8$
5	$1/4$	$1/8$	$1/8$

Find i) the marginal distributions of X and Y

ii) $E(X), E(Y)$ iii) σ_X , σ_Y iv) $Cov(X,Y)$.

c) The joint probability function of two random variables X and Y is given by $f(x,y)=c(2x+y)$ where x and y can assume all integral values such that $0 \le x \le 2,\ 0 \le y \le 3$ and $f(x,y)=0$ otherwise. **7**

Find i) the value of the constant c,

ii) Marginal Probability distribution of X and Y.

Also check whether X and Y are independent.

UNIT - III

4 a) Derive the Crank-Nicolson implicit formula for the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$. **6**

b) Solve the heat equation $u_t = u_{xx}$ subject to the conditions $u(x,0)=\sin(\pi x),\ \ 0 \le x \le 1;\ \ u(0,t)=0=u(1,t),\ t \ge 0$ using Schmidt method by taking $h=\frac{1}{3}, k=\frac{1}{36}$ up to $t=\frac{1}{18}$. **7**

c) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, $0 \le x \le 1$, subject to the initial conditions $u(x,0) = \sin \pi x$, $u_t(x,0) = 0$ and the boundary conditions $u(0,t) = 0,\ u(1,t) = 0$ taking $h = 0.2$ and $k = 0.2$ up to two time levels. **7**

UNIT - IV

5 a) State and prove polar form of Cauchy-Riemann equations and hence deduce that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0.$ **6**

b) If $f(z)=u+iv$ is an analytic function of z, find $f(z)$ if $u - v = x^3 + 3x^2 y - 3xy^2 - y^3$. **7**

c) Discuss the transformation $w = z^2$. **7**

OR

6 a) Find the orthogonal trajectories of the family of curves $x^3y - xy^3 = c$ where c is a constant. **6**

b) If $f(z)$ is a holomorphic function of z, show that **7**

$$\left\{\frac{\partial}{\partial x}|f(z)|\right\}^2 + \left\{\frac{\partial}{\partial y}|f(z)|\right\}^2 = |f'(z)|^2$$

c) Find the bilinear transformation which maps the points $z_1 = 0,\ z_2 = -i,\ z_3 = -1$ onto the points $w_1 = i,\ w_2 = 1,\ w_3 = 0.$ **7**

UNIT - V

7 a) State and prove Cauchy's theorem. **6**

b) Expand $f(z) = \dfrac{1}{(z+1)(z+3)}$ as a Laurent's series in the region **7**

i) $1 < |z| < 3$ ii) $0 < |z+1| < 2$

c) Evaluate $\displaystyle\int_C \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)(z-2)}\,dz$ where C is the circle $|z| = 3$. **7**

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