U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

June 2025 Semester End Main Examinations

Programme: B.E. — **Semester: IV**
Branch: Mechanical Engineering — **Duration: 3 hrs.**
Course Code: 19MA4BSHEM — **Max Marks: 100**
Course: Higher Engineering Mathematics

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	CO	PO	Marks
1	a)	Find the coefficient of correlation for the following data x: 36, 23, 27, 28, 28, 29, 30, 31, 33, 35 y: 29, 18, 20, 22, 27, 21, 29, 27, 29, 28	1	1	**6**
	b)	In a certain factory turning out razor blades there is a small probability of $\frac{1}{500}$ for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) no defective (ii) at least one (iii) at most two defective blades in a consignment of 10,000 packets.	1	1	**7**
	c)	The average number of acres burned by forest and range fires in a large New Mexico county is 4,300 acres per year, with a standard deviation of 750 acres. The distribution of the number of acres burned is normal. What is the probability that between 2,500 and 4,200 acres will be burned in any given year?	1	1	**7**
		OR			
2	a)	If θ is an angle between the two regression lines then show that $\tan\theta = \frac{1-r^2}{r}\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$. Explain the significance when $r=0$ and $r=\pm 1$.	1	1	**6**
	b)	Fit a second-degree polynomial to the following data x: 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0 y: 1.1, 1.3, 1.6, 2.0, 2.7, 3.4, 4.1	1	1	**7**
	c)	In a normal distribution 31% of the items are under 45 and 8% of the items are over 64. Find the mean and standard deviation.	1	1	**7**

<table>
<tr><td></td><td></td><td></td><td>UNIT-2</td><td></td><td></td><td></td></tr>
<tr><td></td><td>3</td><td>a)</td><td>Find the unique fixed probability vector of the regular stochastic matrix $p = \begin{bmatrix} 0 & 3/4 & 1/4 \\ 1/2 & 1/2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>The joint probability distribution table for two random variables X and Y is as follows.
<table>
<tr><td>X \ Y</td><td>−4</td><td>2</td><td>7</td></tr>
<tr><td>1</td><td>1/8</td><td>1/4</td><td>1/8</td></tr>
<tr><td>5</td><td>1/4</td><td>1/8</td><td>1/8</td></tr>
</table>
Find (i) Marginal distributions of X and Y (ii) COV (X, Y).</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Every year, a man trades his car for a new car. If he has Maruti, he trades it for Santro. However if he has Santro, he is just as likely to trade it for a new Santro as to trade it for a Maruti or an Ambassador. In 2000 he bought his first car which was a Santro. Find the probability that he has
(i) 2002 Santro (ii) 2002 Maruti
(iii) 2003 Ambassador (iv) 2003 Santro</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>4</td><td>a)</td><td>Verify that the matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 0.5 & 0.25 & 0.25 \\ 0 & 1 & 0 \end{bmatrix}$ is a regular stochastic matrix.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>If X and Y are independent random variables with the following respective distribution. Find the joint distribution of X and Y. Also verify that COV (X, Y) = 0.
<table>
<tr><td>x_i</td><td>1/8</td><td>1/4</td></tr>
<tr><td>$f(x_i)$</td><td>1/4</td><td>1/8</td></tr>
</table>
<table>
<tr><td>y_j</td><td>1</td><td>1/8</td><td>1/4</td></tr>
<tr><td>$g(y_j)$</td><td>5</td><td>1/4</td><td>1/8</td></tr>
</table></td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>A students’ study habits are as follows: If he studies one night, he is 70% sure not to study the next night. On the other hand, if he does not study one night, he is 60% sure not to study next night as well. In the long run, how often does he study?</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 3</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Derive Bendre-Schmidt finite difference formula for the solution of one-dimensional heat equation $u_t = c^2 u_{xx}$.</td><td>2</td><td>1</td><td>6</td></tr>
</table>

		b)	Solve $u_{tt} = 25u_{xx}$ at pivotal points given $u(0,t) = u(5,t) = 0$, $u_t(x,0) = 0$ and $u(x,\ 0) = \begin{cases} 20x, & 0 \le x \le 1 \\ 5(5-x) & 1 \le x \le 5 \end{cases}$ by taking $h = 1$ and $k = 0.2$. Carryout the computations up to two-time levels.	2	1	**7**
		c)	Find the numerical solution of the parabolic equation $u_t = u_{xx}$ under the conditions $u(0,t) = 0,\ u(1,t) = 0,\ t \ge 0$ and $u(x,0) = \sin \pi x$, $0 \le x \le 1$ by taking $h = 1/4$ and $k = 1/96$ using Schmidt explicit method up to two time levels.	2	1	**7**
			OR			
6	a)		Derive Crank-Nicolson two level implicit formula for the solution of one-dimensional heat equation $u_t = c^2 u_{xx}$.	2	1	**6**
		b)	Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ with the initial conditions $u(x,0) = sin(\pi x)\,, 0 \le x \le 1,\ u_t(x,0) = 0, 0 \le x \le 1$ and the boundary conditions $u(0,t) = u(1,t) = 0$. Carryout the computations up to two time levels taking $h = 0.25$ and $k = 0.5$.	2	1	**7**
		c)	Find the numerical solution of the heat equation $3u_t = u_{xx}$ when $u(0,t) = 0,\ u(4,t) = 0$ and $u(x,0) = x(4-x)$ by taking $h = 1$ and $k = 1$. Carryout the computations up to two-time levels.	2	1	**7**
			UNIT - 4			
7	a)		Derive Cauchy Riemann equations in Cartesian form.	3	1	**6**
		b)	Discuss the conformal transformation of $w = z + \frac{a^2}{z}, z \neq 0$.	3	1	**7**
		c)	Find the bilinear transformation which maps the points $z = 1\,, i, -1$ into the points $w = 0,\ 1,\ \infty$.	3	1	**7**
			OR			
8	a)		Construct the analytic function if $u + v = \frac{1}{r^2}(\cos 2\theta - \sin 2\theta)$.	3	1	**6**
		b)	If $f(z)$ is analytic, then show that $\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right] \lvert f(z)\rvert^2 = 4\left\lvert f'(z)\right\rvert^2$.	3	1	**7**
		c)	Find the bilinear transformation which maps the points $z = \infty\,, i, 0$ into the points $w = -1\,, -i, 1$.	3	1	**7**
			UNIT - 5			
9	a)		Evaluate $\int_C \frac{e^{2z}}{(z+1)^2(z-2)} dz$ where C is the circle $\lvert z\rvert = 3$.	3	1	**6**
		b)	Obtain the Laurent series expansion of $\frac{e^z}{(z-1)(z-3)}$ in the region $1 < \lvert z\rvert < 3$.	3	1	**7**

		c)	Apply Cauchy's residue theorem to evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z}{(z-1)^2 (z-2)} dz$ where C is the circle $\lvert z \rvert = 3$.	3	1	**7**
			OR			
	10	a)	State and prove the Cauchy's integral formula.	3	1	**6**
		b)	Verify Cauchy's theorem for the integral of $\frac{1}{z}$ taken over the boundary of the triangle having vertices $(1,2), (1,4)$ and $(3,2)$.	3	1	**7**
		c)	Find the Laurent's expansion of $f(z) = \frac{7z-2}{(z+1)z(z-2)}$ in the region $1 < z+1 < 3$.	3	1	**7**

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