U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

February 2025 Semester End Main Examinations

Programme: B.E. **Semester: IV**
Branch: CS cluster except AIML **Duration: 3 hrs.**
Course Code: 23MA4BSLAO **Max Marks: 100**
Course: Linear Algebra and Optimization

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT – 1	***CO***	***PO***	**Marks**
1	a)	Find the gradient of matrix $\boldsymbol{f} = \begin{bmatrix} \sin(x_0 + 2x_1) & 2x_1 + x_3 \\ 2x_0 + x_2 & \cos(2x_2 + x_3) \end{bmatrix}$ with respect to the matrix $\boldsymbol{x} = \begin{bmatrix} x_0 & x_1 \\ x_2 & x_3 \end{bmatrix}$ and hence verify $\frac{\partial\,[\mathrm{Trace}(\boldsymbol{f}(\boldsymbol{x}))]}{\partial \boldsymbol{x}} = \mathrm{Trace}\left(\frac{\partial \boldsymbol{f}(\boldsymbol{x})}{\partial \boldsymbol{x}}\right)$.	*1*	*1*	**6**
	b)	Given $f(x,y,z) = -x^3 + 3xz + 2y - y^2 - 3z^2$, i) find all the stationary points of the function, ii) find the Hessian matrix and iii) classify the stationary points and find its extreme value.	*1*	*1*	**7**
	c)	Show that $S = \{(x_1, x_2, x_3): x_1^2 + x_2^2 + x_3^2 \le 1\}$ is a convex set.	*1*	*1*	**7**
		OR			
2	a)	Obtain the gradient of the matrix $\begin{bmatrix} xy & yz & zw \\ wx & wy & wz \end{bmatrix}$ with respect to $\begin{bmatrix} x & y \\ z & w \end{bmatrix}$ and hence verify $\frac{\partial (f(X)^T}{\partial X} = \left(\frac{\partial f(X)}{\partial X}\right)^T$.	*1*	*1*	**6**
	b)	Given $f(x,y,z) = x^2 + y^2 + z^2 - 2xy + 2yx - 2zy$, i) find all the stationary points of the function, ii) find the Hessian matrix and iii) classify the stationary points and find its extreme value.	*1*	*1*	**7**
	c)	Let $A = \left\{\begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 6 \\ 0 \end{bmatrix}\right\}$, $B = \left\{\begin{bmatrix} 0 \\ 5 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}\right\}$, and $\boldsymbol{n} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$. Find a hyperplane H with normal $\boldsymbol{n}$ that separates A and B.	*1*	*1*	**7**
		UNIT – 2			
3	a)	Apply Newton's method to find the minimum value of the function $f = 4x_1^2 + x_2^2 - 2x_1x_2$ near $(1, 1)$.	*1*	*1*	**6**
	b)	Apply Fibonacci search algorithm to minimize $f(x) = x[x - 1.5]$ in $[0, 1]$ within the interval of uncertainty 0.25 of the initial interval of uncertainty.	*1*	*1*	**7**

		c)	Derive the KKT conditions to minimize the function $f(x,y)=xy$ subject to the constraints $x+y^2\le 2$ and $x,y\ge 0$. Also find the optimum values of the function.	*1*	*1*	**7**
			OR			
	4	a)	Approximate the minimum point of the function $f(x,y)=4x^2-8xy+6y^2$ near $(1,1)$ using Gradient descent/ascent method. Perform three iterations.	*1*	*1*	**6**
		b)	An asteroid is entering the atmosphere of moon. The shape of the asteroid is described by the equation $4x^2+y^2+4z^2=16$. The temperature on the surface of the asteroid after one month was observed to be represented by equation $8x^2+4yz-16z+600$. Is it possible to find the point on surface of asteroid with maximum temperature? If yes, find it?	*1*	*1*	**7**
		c)	Derive the KKT conditions to maximize $-(x-2)^2-2(y-1)^2$ subject to $x+4y\le 3$ and $x\ge y$.	*1*	*1*	**7**
			UNIT – 3			
	5	a)	Consider $A=\begin{bmatrix}1&2&1\\2&5&4\\1&4&6\end{bmatrix}$ and the vectors $u=\begin{bmatrix}1\\\alpha\\1\end{bmatrix}$ and $v=\begin{bmatrix}2\\2\\3\end{bmatrix}$. The inner product is defined as $\langle u,v\rangle=u^TAv$. Find the value of α if it is known that the vectors u and v are orthogonal and hence find the length of u with respect to the given inner product.	*1*	*1*	**6**
		b)	Solve the system of equations $AX=B$ where $A=\begin{bmatrix}2&3\\2&4\\1&1\end{bmatrix}$ and $B=\begin{bmatrix}7\\3\\1\end{bmatrix}$ by QR factorization method.	*1*	*1*	**7**
		c)	Fit a parabola of the form $y=a+bx$ for the data (1, 1), (2, 4), (3, 7) and (4, 5) using the method of least squares.	*1*	*1*	**7**
			OR			
	6	a)	Find the angle between the vectors $A=\begin{bmatrix}9&8&7\\6&5&4\end{bmatrix}$ and $B=\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}$ where $\langle A,B\rangle=Tr\left(B^TA\right)$.	*1*	*1*	**6**
		b)	Let $P_2(t)$ be the vector space of polynomials of degree ≤ 2 with $\langle f,\ g\rangle=\int_0^1 f(t)g(t)dt$. Find a basis of the subspace W orthogonal to $h(t)=2t+1$.	*1*	*1*	**7**
		c)	Apply the Gram-Schmidt orthogonalization to find the orthonormal basis of the subspace $W\subseteq\mathbb{R}^3$ spanned by the vectors $v_1=(1,-1,1)$, $v_2=(1,\ 2,3)$ and $v_3=(5,\ 6,\ 7)$.	*1*	*1*	**7**

			UNIT – 4			
	7	a)	Apply Cayley-Hamilton theorem to find A^4 given $A=\begin{bmatrix}1&2&3\\2&4&5\\3&5&6\end{bmatrix}$.	*1*	*1*	**6**
		b)	Find an eigenspace of the linear transformation $T:P_2(t)\to P_2(t)$ defined by $T\left(at^2+bt+c\right)=(2a-c)t^2+(2a+b-2c)t+(-a+2c)$.	*1*	*1*	**7**
		c)	Find the characteristic and minimal polynomial of $\begin{bmatrix}2&5&0&0&0\\0&2&0&0&0\\0&0&4&2&0\\0&0&3&5&0\\0&0&0&0&7\end{bmatrix}$.	*1*	*1*	**7**
			OR			
	8	a)	Apply Cayley-Hamilton theorem to find A^{-3} if $A=\begin{bmatrix}4&6&6\\1&3&2\\-1&-4&-3\end{bmatrix}$.	*1*	*1*	**6**
		b)	Obtain the algebraic and geometric multiplicity of the eigenvalues for the matrix $A=\begin{bmatrix}2&0&1\\0&1&0\\1&0&2\end{bmatrix}$ by finding the corresponding eigenspaces.	*1*	*1*	**7**
		c)	Find the Jordan canonical form of the linear transformation defined by the matrix $A=\begin{bmatrix}1&2&-1\\0&2&0\\1&-2&3\end{bmatrix}$.	*1*	*1*	**7**
			UNIT - 5			
	9	a)	Find the nature of the quadratic form $x^2+5y^2+z^2+2xy+6xz+2yz$ and hence write its canonical form.	*1*	*1*	**5**
		b)	Determine a singular value decomposition of $A=\begin{bmatrix}3&2\\2&3\\2&-2\end{bmatrix}$.	*1*	*1*	**8**
		c)	Suppose three tests are administered to a random sample of college students. Let $\boldsymbol{X}_1 \ldots \boldsymbol{X}_N$ be observation vectors in R^3 that list the three scores of each student, and for $j=1,2$ and 3, let x_j denote a student's score on the j^{th} exam. The covariance matrix of the above data is $S=\begin{bmatrix}5&2&0\\2&6&2\\0&2&7\end{bmatrix}$. Let y be an "index" of student performance, with $y=c_1x_1+c_2x_2+c_3x_3$ and $c_1^2+c_2^2+c_3^2=1$. Compute c_1, c_2 and c_3 so that the variance of y over the data set is as large as possible using principal component analysis.	*1*	*1*	**7**
			OR			

	10	a)	Determine the modal matrix that reduces the quadratic form $x^2+5y^2+z^2+2xy+6xz+2yz$ to its canonical form and hence discuss the nature the quadratic form.	*1*	*1*	**10**
		b)	Reduce the dimension from two to one using principal component analysis for the following data: x: 4, 8, 13, 7 y: 11, 4, 5, 14	*1*	*1*	**10**

x	4	8	13	7
y	11	4	5	14

******