U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

October 2024 Supplementary Examinations

Programme: B.E. **Semester: IV**
Branch: CS cluster except AIML **Duration: 3 hrs.**
Course Code: 23MA4BSLAO **Max Marks: 100**
Course: Linear Algebra and Optimization

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

			CO	*PO*	**Marks**
		UNIT – 1			
1	a)	Find the gradient of matrix $\boldsymbol{f} = \begin{bmatrix} x_0^2 x_1 \log(x_2) & \frac{x_1^2 x_2}{x_3} \\ x_3^2 + x_1 x_3 & x_2^3 + x_1 \end{bmatrix}$ with respect to the matrix $\boldsymbol{x} = \begin{bmatrix} x_0 & x_1 \\ x_2 & x_3 \end{bmatrix}$ and hence verify $\frac{\partial\,[\mathrm{Trace}(\boldsymbol{f}(\boldsymbol{x}))]}{\partial \boldsymbol{x}} = \mathrm{Trace}\left(\frac{\partial \boldsymbol{f}(\boldsymbol{x})}{\partial \boldsymbol{x}}\right)$.	*1*	*1*	**7**
	b)	Given $f = x^2 + y^2 + z^2 + xy + yz + zx - 4x - 4y - 4z - 5$, i) find all the stationary points of the function, ii) find the Hessian matrix and iii) classify the stationary points and find its extreme value.	*1*	*1*	**7**
	c)	Find the equation of hyperplane normal to the vector $\boldsymbol{n} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$ that separates the sets $A = \left\{ \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 6 \\ 0 \end{bmatrix} \right\}$ and $B = \left\{ \begin{bmatrix} 0 \\ 5 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \right\}$. Does this hyperplane strictly separate these sets?	*1*	*1*	**6**
		UNIT – 2			
2	a)	Apply Newtons method to find the minimum value of the function $f = 4x_1^2 + x_2^2 - 2x_1x_2 + x_1 + x_2$ near $(1, 1)$. Carry out two iterations.	*1*	*1*	**6**
	b)	An asteroid is entering the atmosphere of moon. The shape of the asteroid is described by the equation $4x^2 + y^2 + 4z^2 = 16$. The temperature on the surface of the asteroid after one month was observed to be represented by equation $8x^2 + 4yz - 16z + 600$. Is it possible to find the point on surface of asteroid with maximum temperature? If yes, find it?	*1*	*1*	**6**
	c)	Derive the KKT conditions to minimize the function $f(x, y) = xy$ subject to the constraints $x + y^2 \le 2$ and $x, y \ge 0$. Also find the optimum values of the function.	*1*	*1*	**8**

<table>
<tr><td></td><td></td><td></td><td>UNIT – 3</td><td></td><td></td><td></td></tr>
<tr><td></td><td>3</td><td>a)</td><td>If $u=(1,\ 3,\ -4,\ 2)$, $v=(4,-2,2,1)$ and $w=(5,-1,-2,6)$ are vectors in $\mathbb{R}^4$ then:
i) verify the $\langle 3u-2v, w\rangle = 3\langle u,w\rangle - 2\langle v,w\rangle$ and
ii) find $\frac{\|3u-2v\|}{\|w\|}$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find an orthogonal basis of the subspace W spanned by the following vectors $S=\{1,\ t,\ t^2\}$ of $P_2(t)$ given $\langle f,g\rangle = \int_0^1 f(t)g(t)\,dt$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>A sales organization obtains the following data relating the number of salespersons to annual sales.
<table>
<tr><td>x: Number of salespersons</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>y:Annual Sales (millions of dollars)</td><td>2.3</td><td>3.2</td><td>4.1</td><td>5.0</td><td>6.1</td><td>7.2</td></tr>
</table>
Find the least squares line of the form $y = a + bx$ and hence estimate the annual sales when there are 14 salespersons.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>4</td><td>a)</td><td>Find the value of α such that the matrices $A = \begin{bmatrix} \alpha & 8 & -7 \\ 6 & 5\alpha & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 5 & 6\alpha \end{bmatrix}$ are orthogonal with respect to an inner product $\langle A,B\rangle = Tr(B^T A)$. Hence find $\|A\|$ and $\|B\|$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Show that $S=\{u_1,u_2,u_3,u_4\}$ where $u_1=(1,1,1,1)$, $u_2=(1,1,-1,-1)$, $u_3=(1,-1,1,-1)$, $u_4=(1,-1,-1,1)$ is orthogonal and a basis of $\mathbb{R}^4$.
Find the coordinates of the arbitrary vector $v=(a,b,c,d)$ in $\mathbb{R}^4$ relative to the basis S.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Obtain the QR factorization of the following matrix $A=\begin{bmatrix} 1 & -1 \\ 2 & 0 \\ 3 & 1 \end{bmatrix}$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 4</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Apply Cayley-Hamilton theorem to find A^4 given $A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Given $M = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ and eigenvalues of M are $\lambda = 0, 2$. Find the eigenvalues and the eigenspace corresponding to each eigenvalue of $A = M^2 + \frac{1}{2}M$. Hence determine whether A is defective matrix or not.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Determine all possible Jordan canonical forms or blocks of linear transformation T with characteristic polynomial $f(t)=(t-4)^3$ and specify the geometric multiplicity in each case.</td><td>1</td><td>1</td><td>7</td></tr>
</table>

			OR			
	6	a)	Determine the inverse of the matrix $A=\begin{bmatrix}4 & 6 & 6\\ 1 & 3 & 2\\ -1 & -4 & -3\end{bmatrix}$ by using Cayley-Hamilton theorem.	*1*	*1*	**6**
		b)	Find an eigenspace of the linear transformation $T:P_2(t)\rightarrow P_2(t)$ given by $T\left(at^2+bt+c\right)=(a+3b+3c)t^2-(3a+5b+3c)t+(3a+3b+c)$.	*1*	*1*	**7**
		c)	Find the characteristic and minimal polynomial of the matrix $A=\begin{bmatrix}2 & 5 & 0 & 0 & 0\\ 0 & 2 & 0 & 0 & 0\\ 0 & 0 & 4 & 2 & 0\\ 0 & 0 & 3 & 5 & 0\\ 0 & 0 & 0 & 0 & 7\end{bmatrix}$.	*1*	*1*	**7**
			UNIT - 5			
	7	a)	Find the nature and write the canonical form of the quadratic form $3x^2+3y^2+3z^2+2xy+2xz-2yz$.	*1*	*1*	**5**
		b)	Assume that the **USN** is **XYZ2222** and then construct a matrix **A** of size 2×2 by taking last four digits of the **USN** given and hence find the singular value decomposition. Also write comment on the dimension reduction.	*1*	*1*	**8**
		c)	Apply principal component analysis to given data to reduce from two-dimension to 1-dimension: x: 4, 8, 13, 7 y: 11, 4, 5, 14	*1*	*1*	**7**

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