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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

June 2025 Semester End Main Examinations

Programme: B.E. **Semester: IV**
Branch: CS Cluster Except AIML and CSBS **Duration: 3 hrs.**
Course Code: 23MA4BSLAO **Max Marks: 100**
Course: Linear Algebra and Optimization

Instructions 1. Each unit has an internal choice. Answer one complete question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	*CO*	*PO*	**Marks**
1	a)	Let $v_1 = \begin{bmatrix}1\\1\\3\end{bmatrix}$, $v_2 = \begin{bmatrix}2\\4\\1\end{bmatrix}$ and $v_3 = \begin{bmatrix}-1\\-2\\5\end{bmatrix}$. Find an implicit description of hyperplane H that passes through v_1, v_2, and v_3.	*1*	*1*	**6**
	b)	Obtain the critical points of $f(x,y,z) = xy + yz + zx - 4x + 2y$ and hence find the nature of the critical points of f using the Hessian matrix.	*1*	*1*	**7**
	c)	Given $\boldsymbol{f} = \begin{bmatrix}x_0^2x_1x_2 & x_1^2x_2x_3\\ x_3^2x_1x_2 & x_2^3x_1\end{bmatrix}$ and $\boldsymbol{X} = \begin{bmatrix}x_0 & x_1\\ x_2 & x_3\end{bmatrix}$, then verify the identity $\frac{\partial\left(\text{Trace}(\boldsymbol{f}(\boldsymbol{X}))\right)}{\partial \boldsymbol{X}} = \text{Trace}\left(\frac{\partial \boldsymbol{f}(\boldsymbol{X})}{\partial \boldsymbol{X}}\right)$.	*1*	*1*	**7**
		OR			
2	a)	Show that the set $S = \{(x,y) \mid 2x + 3y = 5\} \subset R^2$ is a convex set.	*1*	*1*	**6**
	b)	Find the value of 'a' for which $P = (0,0,0)$ is the critical point of the function $f(x,y,z) = 2x^2 + 3y^2 + 4z^2 + axyz$ and hence find the nature of the critical point P using the Hessian matrix.	*1*	*1*	**7**
	c)	If $\boldsymbol{f} = \begin{bmatrix}\sin(x_0 + 2x_1) & 2x_1 + x_3\\ 2x_0 + x_2 & \cos(2x_2 + x_3)\end{bmatrix}$ and $\boldsymbol{X} = \begin{bmatrix}x_0 & x_1\\ x_2 & x_3\end{bmatrix}$ then verify the identity $\frac{\partial (f(\boldsymbol{X})^T)}{\partial \boldsymbol{X}} = \left(\frac{\partial f(\boldsymbol{X})}{\partial \boldsymbol{X}}\right)^T$.	*1*	*1*	**7**
		UNIT - 2			
3	a)	Minimize $f(x) = x^2 - 2.6x + 2, x \in [-2,3]$ using Fibonacci search method. Perform six iterations.	*1*	*1*	**6**
	b)	Find the maximum and minimum values of $f(x,y,z) = 4y - 2z$ subjected to the constraints $2x - y - z = 2$ and $x^2 + y^2 = 1$ using method of Lagrange multipliers.	*1*	*1*	**7**
	c)	Maximize $Z = 12x_1 + 21x_2 + 2x_1x_2 - 2x_1^2 - 2x_2^2$ subjected to the constraints $x_1 + x_2 \leq 10,\ x_2 \leq 8,\ x_1, x_2 \geq 0$ by deriving the KKT conditions.	*1*	*1*	**7**
		OR			

	4	a)	Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ using Newton-Raphson method by taking initial point as $\begin{bmatrix}0\\0\end{bmatrix}$.	*1*	*1*	**6**
		b)	Find the maximum and minimum values of $f(x,y,z) = x + y + z^2$ subjected to the constraints $x + y + z = 1$ and $x^2 + z^2 = 1$ using the method of Lagrange multipliers.	*1*	*1*	**7**
		c)	Minimize $Z = (x-3)^2 + (y-2)^2$ subjected to the constraints $x^2 + y^2 \leq 5,\ x + 2y \leq 4$ and $x, y \geq 0$ by deriving the KKT conditions.	*1*	*1*	**7**
			UNIT - 3			
	5	a)	Let W be a subspace of $\mathbb{R}^5$ spanned by $u = (1,2,3,-1,2)$ and $v = (2,4,7,2,-1)$. Find the basis of $W^{\perp}$.	*1*	*1*	**6**
		b)	Find an orthogonal matrix P whose first row is $u_1 = \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$.	*1*	*1*	**7**
		c)	Solve the system of equations $AX = B$ where $A = \begin{bmatrix}2 & 3\\2 & 4\\1 & 1\end{bmatrix}$ and $B = \begin{bmatrix}7\\3\\1\end{bmatrix}$ by QR factorization method.	*1*	*1*	**7**
			OR			
	6	a)	Consider $f(t) = 3t - 5$, $g(t) = t^2$ in $P(t)$ and the inner product is defined as $\langle p, q\rangle = \int_0^1 p(t)q(t)dt$. Find $\langle f, g\rangle$, $\lVert f\rVert$ and $\lVert g\rVert$.	*1*	*1*	**6**
		b)	Find the projection of $v = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ along $w = \begin{bmatrix}1 & 1\\5 & 5\end{bmatrix}$ in $M_{2\times 2}$ with respect to the inner product $\langle A, B\rangle = tr(B^T A)$.	*1*	*1*	**7**
		c)	A sales organization obtains the following data relating the number of salespersons to annual sales. <table><tr><td>Number of salespersons</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr><tr><td>Annual Sales (millions of dollars)</td><td>2.3</td><td>3.2</td><td>4.1</td><td>5.0</td><td>6.1</td><td>7.2</td></tr></table>Let x denote the number of salespersons and let y denote the annual sales. Find the least squares line of the form $y = a + bx$ and hence estimate the annual sales when there are 14 salespersons.	*1*	*1*	**7**
			UNIT - 4			
	7	a)	Given $B = \begin{bmatrix}1 & 1 & 0\\1 & 2 & 0\\0 & 0 & 1\end{bmatrix}$, find B^6 using Cayley-Hamilton theorem with minimal conventional operations.	*1*	*1*	**6**
		b)	Find the algebraic and geometric multiplicities of each eigenvalue λ of the linear transformation $T: R^3 \to R^3$ defined by $T(x,y,z) = (x - y + z, x - y + z, x - 2y + 2z)$.	*1*	*1*	**7**
		c)	Let $\Delta(t) = (t+6)^6$ and $m(t) = (t+6)^3$ be the characteristic and minimal polynomial of T respectively. Obtain all possible Jordan canonical forms of T.	*1*	*1*	**7**
			OR			

	8	a)	Given $B = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$, find B^{-3} using Cayley-Hamilton theorem with minimal conventional operations.	*1*	*1*	**6**
		b)	Find the eigenspace corresponding to each eigenvalue of $T: P_1 \to P_1$ defined by $T(at+b) = (3a+5b)t - (2a+4b)$.	*1*	*1*	**7**
		c)	Find the characteristic and minimal polynomial of the matrix $\begin{bmatrix} 2 & 5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 3 & 5 & 0 \\ 0 & 0 & 0 & 0 & 7 \end{bmatrix}$.	*1*	*1*	**7**
			UNIT - 5			
	9	a)	Obtain the transformation matrix that reduces the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ to its canonical form and hence find the rank and index of the quadratic form.	*1*	*1*	**10**
		b)	Find the singular value decomposition of the matrix $\begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$.	*1*	*1*	**10**
			OR			
	10	a)	Find the orthogonal modal matrix that diagonalizes the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$.	*1*	*1*	**10**
		b)	Apply principal component analysis to reduce the given 2-dimensional data to 1-dimensional data. x: 1, 5, 2, 6, 7, 3 y: 3, 11, 6, 8, 15, 11	*1*	*1*	**10**

x	1	5	2	6	7	3
y	3	11	6	8	15	11

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