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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

September / October 2023 Supplementary Examinations

Programme: B.E	**Semester: IV**
Branch: CS/IS/AIML	**Duration: 3 hrs.**
Course Code: 19MA4BSLIA	**Max Marks: 100**
Course: Linear Algebra	**Date: 14.09.2023**

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

UNIT - I

1 a) Apply LU decomposition method to solve the system of equations $2x+y+4z=12,\ 4x+11y-z=33$ and $8x-3y+2z=20$. **6**

b) Express the polynomial $v=t^2+4t-3$ in $p(t)$ as a linear combination of the polynomials $p_1=t^2-2t+5$, $p_2=2t^2-3t$, $p_3=t+3$. **7**

c) Let W be a subspace of $\mathbb{R}^4$ spanned by the vectors $u_1=(1,-2,5,-3)$, $u_2=(2,3,1,-4)$ and $u_3=(3,8,-3,-5)$. **7**
   i) Find the basis and dimension of W.
   ii) Extend the basis of W to a basis of $\mathbb{R}^4$.

OR

2 a) Show that the vectors $u_1=(1,1,1),\ u_2=(1,2,3),\ u_3=(1,5,8)$ span $\mathbb{R}^3$. **6**

b) Show that the set $V=\left\{a+b\sqrt{2}\mid a,b\in\mathbb{Q}\right\}$ is a vector space over the field of rationals. **7**

c) Find the dimension and a basis of the solution space W of the system of equations $x+2y+2z-s+3t=0$; $x+2y+3z+s+t=0$ and $3x+6y+8z+s+5t=0$. **7**

UNIT- II

3 a) If $T:\mathbb{R}^3\to\mathbb{R}^3$ defined by $T(x,y,z)=(x-y,y+z,x+z)$, find $R(T)$, $N(T)$ and hence verify Rank-Nullity theorem. **6**

b) Let T be a linear operator defined on $\mathbb{R}^3$ through $T(x,y,z)=(2x+3y-z,4y-z,2z)$. Is T invertible? If so, then find a formula for T^{-1}. **7**

c) Find the matrix of linear transformation $T:V_2(\mathbb{R})\to V_3(\mathbb{R})$ defined by $T(x,y)=(-x+2y,y,-3x+3y)$ relative to the basis $B_1=\{(1,1),(-1,1)\}$ and $B_2=\{(1,1,1),(1,-1,1),(0,0,1)\}$. **7**

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

UNIT- III

4 a) Find A^4 given $A=\begin{bmatrix}1&2&3\\2&4&5\\3&5&6\end{bmatrix}$ using Cayley-Hamilton theorem. **6**

b) Find the eigen space of the linear transformation $T:R^3\to R^3$ defined by $T(x,\ y,\ z)=(3x+2y+z,\ x+4y+z,\ 2y+4z)$. **7**

c) Determine all possible Jordan canonical forms of the linear operator $T:V\to V$ whose characteristic polynomial is $\Delta(t)=(t-2)^3(t-5)^2$. **7**

OR

5 a) Find the characteristic and minimal polynomial of $A=\begin{bmatrix}3&2&0&0&0\\1&4&0&0&0\\0&0&3&1&0\\0&0&1&3&0\\0&0&0&0&4\end{bmatrix}$. **6**

b) Find eigenvalues and eigenvectors of the linear transformation $T:V_2(\mathbb{R})\to V_2(\mathbb{R})$ defined by $T(x,\ y)=(2x+5y,\ 4x+3y)$. **7**

c) Express the initial-value problem $\frac{d^2x(t)}{dt^2}-2\frac{dx(t)}{dt}-3x(t)=0$ subjected to $x(0)=4,\ \frac{dx(0)}{dt}=5$ into fundamental form and hence solve. **7**

UNIT- IV

6 a) Let W be a subspace of $\mathbb{R}^5$ spanned by $u=(1,2,3,-1,2)$ and $v=(2,4,7,2,-1)$. Find a basis of the orthogonal compliment $W^{\perp}$ of W. **6**

b) Find an orthogonal basis and hence an orthonormal basis of U, the subspace of $\mathbb{R}^4$ spanned by the vectors $v_1=(1,1,1,1)$, $v_2=(1,1,2,4)$ and $v_3=(1,2,-4,-3)$ using Gram-Schmidt orthogonalization process. **7**

c) Obtain the QR factorization of $A=\begin{bmatrix}1&1&2\\0&0&1\\1&0&0\end{bmatrix}$. **7**

UNIT- V

7 a) Orthogonally diagonalize $\begin{bmatrix}3&-2&4\\-2&6&2\\4&2&3\end{bmatrix}$. **10**

b) Find a singular value decomposition of $\begin{bmatrix}1&-1\\-2&2\\2&-2\end{bmatrix}$. **10**

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