U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

December 2023 Supplementary Examinations

Programme: B.E. | **Semester: IV**
Branch: CS, IS and AI&ML | **Duration: 3 hrs.**
Course Code: 22MA4BSLIA | **Max Marks: 100**
Course: Linear Algebra

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

			CO	***PO***	**Marks**
		UNIT - I			
1	a)	Show that the set $M=\left\{\begin{bmatrix}0 & a\\ b & 0\end{bmatrix} \Big/ a,b\in\mathbb{R}\right\}\subset M_{2\times2}$ satisfies all the properties of a vector space over the field of reals under standard matrix addition and scalar multiplication.	*CO1*	*PO1*	**6**
	b)	Does there exist non-zero scalars c_1, c_2, c_3 and c_4 which proves that the vectors $v_1=(0,1,2,3,0)$, $v_2=(1,3,-1,2,1)$, $v_3=(2,6,-1,-3,1)$ and $v_4=(4,0,1,0,2)$ in $\mathbb{R}^5$ are linearly dependent? If yes, find them.	*CO1*	*PO1*	**7**
	c)	Determine a subset of $S=\{p_1,p_2,p_3,p_4\}\subset P_3(t)$, the vector space of polynomials that forms a basis of $W=span(S)$ if $p_1=t^3+t^2$, $p_2=2t^3+2t-2$, $p_3=t^3-6t^2+3t-3$ and $3t^2-t+1$.	*CO1*	*PO1*	**7**
		UNIT – II			
2	a)	Consider the matrix $A=\begin{bmatrix}2 & 4\\ 5 & 6\end{bmatrix}$ which defines a linear operator on $\mathbb{R}^2$. Find the matrix of the linear transformation relative to the basis $S=\{u_1,u_2\}=\{(1,-2),(3,-7)\}$.	*CO1*	*PO1*	**6**
	b)	Find the basis for the range space $R(T)$, null space $N(T)$ for the linear transformation $T:\mathbb{R}^4\to\mathbb{R}^3$ given by $T(x,y,z,t)=(x-y+z+t,\ x+2z+t,\ x+y+3z-3t)$ and also verify rank-nullity theorem.	*CO1*	*PO1*	**7**
	c)	Let $G:\mathbb{R}^3\to\mathbb{R}^3$ given by $G(x,y,z)=(x+y,\ x+z,\ y+z)$. (i) Show that G is invertible. (ii) Find G^{-1}.	*CO1*	*PO1*	**7**
		OR			

	3	a)	Let $A=\begin{bmatrix}1 & -3\\ 3 & 5\\ -1 & 7\end{bmatrix}$, $u=\begin{bmatrix}2\\ -1\end{bmatrix}$, $b=\begin{bmatrix}3\\ 2\\ -5\end{bmatrix}$ and $c=\begin{bmatrix}3\\ 2\\ 5\end{bmatrix}$. Define a linear transformation $T:\mathbb{R}^2\rightarrow\mathbb{R}^3$ by $T(X)=AX$, for $X\in\mathbb{R}^2$. (i) Find the image of u under the transformation T. (ii) Find an $X\in\mathbb{R}^2$ whose image is b. Is there more than one $X\in\mathbb{R}^2$ whose image is b? (iii) Determine if c is in the range of the transformation.	*CO1*	*PO1*	**6**
		b)	Derive the matrix of the linear transformation $T:\mathbb{R}^2\rightarrow\mathbb{R}^2$, which results in horizontal shear of $(x,y)\in\mathbb{R}^2$ by 0.5 units. Determine if there exist i) a preimage of $(1,-3)$. ii) an image of $(3,-1)$.	*CO1*	*PO1*	**7**
		c)	Find the basis for the range space $R(T)$ and the Kernel $N(T)$ of the linear transformation $T:\mathbb{R}^4\rightarrow\mathbb{R}^3$ given by the matrix $A=\begin{bmatrix}1 & 2 & 3 & 1\\ 1 & 3 & 5 & -2\\ 3 & 8 & 13 & -3\end{bmatrix}$. Hence verify the rank-nullity theorem. Is T a one-one mapping? Justify.	*CO1*	*PO1*	**7**
			UNIT - III			
	4	a)	Apply Cayley-Hamilton theorem to compute A^{-1} of $A=\begin{bmatrix}1 & 1 & -2\\ -1 & 2 & 1\\ 0 & 1 & -1\end{bmatrix}$.	*CO1*	*PO1*	**6**
		b)	Find the eigenvalues and the eigenvectors of the linear transformation $T:P_1(t)\rightarrow P_1(t)$ defined $T(at+b)=(a+2b)t+(4a+3b)$.	*CO1*	*PO1*	**7**
		c)	Find the characteristic and minimal polynomial of $\begin{bmatrix}2 & 5 & 0 & 0 & 0\\ 0 & 2 & 0 & 0 & 0\\ 0 & 0 & 4 & 2 & 0\\ 0 & 0 & 3 & 5 & 0\\ 0 & 0 & 0 & 0 & -7\end{bmatrix}$.	*CO1*	*PO1*	**7**
			OR			
	5	a)	Apply Cayley-Hamilton theorem to compute A^4 if $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\\ 3 & 5 & 6\end{bmatrix}$.	*CO1*	*PO1*	**6**
		b)	Determine the eigenspaces of the linear transformation $T:\mathbb{R}^3\rightarrow\mathbb{R}^3$ defined by $T(x,y,z)=(x+y-z,\,0\,,\,x+2y+3z)$.	*CO1*	*PO1*	**7**

<table>
<tr><td></td><td></td><td>c)</td><td>Write all possible Jordan canonical form of the linear transformation $T:\mathbb{R}^3 \to \mathbb{R}^3$ when $\Delta(t)=(t+5)^2(t-7)^3$ is the characteristic polynomial.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT – IV</td><td></td><td></td><td></td></tr>
<tr><td></td><td>6</td><td>a)</td><td>Find a basis of W of $\mathbb{R}^4$ orthogonal to $u_1=(1,-2,3,4)$ and $u_2=(3,-5,7,8)$.</td><td>CO1</td><td>PO1</td><td>4</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find an orthogonal basis and hence an orthonormal basis of the subspace W spanned by $S=\{1,\ 1-t,\ t^2\}$ of $P_2(t)$ given $\langle f,g\rangle=\int_0^1 f(t)g(t)\,dt$.</td><td>CO1</td><td>PO1</td><td>9</td></tr>
<tr><td></td><td></td><td>c)</td><td>In an experiment designed to determine the extent of a person's natural orientation, a subject is put in a special room and kept there for a certain length of time. He is then asked to find a way of a maze and record is made of the time it takes the subject to accomplish this task. The following data are obtained.
<table>
<tr><td>Time in Room (hours)</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>Time to find way out of maze(minutes)</td><td>0.8</td><td>2.1</td><td>2.6</td><td>2.0</td><td>3.1</td><td>3.3</td></tr>
</table>
Let x denote the number of hours in the room and let y denote the number of minutes that it takes the subject to find his way out.
i. Find the least squares line of the form $y=a+bx$.
ii. Estimate the time it will take the subject to find his way out of the maze after 10 hours in the room using the equation obtained.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT – V</td><td></td><td></td><td></td></tr>
<tr><td></td><td>7</td><td>a)</td><td>Determine the modal matrix that reduces the quadratic form $x^2+3y^2+3z^2-2yz$ to its canonical form and hence discuss the nature the quadratic form.</td><td>CO1</td><td>PO1</td><td>10</td></tr>
<tr><td></td><td></td><td>b)</td><td>Obtain the singular value decomposition of $A=\begin{bmatrix}-3 & 1\\ 6 & -2\\ 6 & -2\end{bmatrix}$.</td><td>CO1</td><td>PO1</td><td>10</td></tr>
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