U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

February 2025 Semester End Main Examinations

Programme: B.E. **Semester:**

Branch: CS / IS / AI and ML **Duration: 3 hrs.**

Course Code: 22MA4BSLIA **Max Marks: 100**

Course: Linear Algebra

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	*CO*	*PO*	**Marks**
1	a)	Determine whether or not W is a subspace of R^3 where W consists of all vectors of the form (a,b,c) in R^3 such that: i. $a+b+c=0$, ii. $a^2+b^2+c^2 \le 1$.	*1*	*1*	**6**
	b)	Express $M=\begin{bmatrix}4&7\\7&9\end{bmatrix}$ as a linear combination of $A=\begin{bmatrix}1&1\\1&1\end{bmatrix}$, $B=\begin{bmatrix}1&2\\3&4\end{bmatrix}$ and $C=\begin{bmatrix}1&1\\4&5\end{bmatrix}$.	*1*	*1*	**7**
	c)	Determine a subset of $S=\{u_1, u_2, u_3, u_4\}$ that gives a basis for $W=span(u_i)$ of $\mathbb{R}^5$ where $u_1=(1,-2,1,3,-1)$, $u_2=(-2,4,-2,-6,2)$, $u_3=(1,-3,1,2,1)$ and $u_4=(3,-7,3,8,-1)$.	*1*	*1*	**7**
		OR			
2	a)	Let $\mathbb{R}^+$ be the set of all positive real numbers. Define vector addition as $u+v=uv$, $\forall\, u,v\in\mathbb{R}^+$ and scalar multiplication $k.u=u^k$ $\forall\, k\in\mathbb{R}$. Show that $\mathbb{R}^+$ is a vector space over the field of real numbers.	*1*	*1*	**6**
	b)	Find the basis and dimension of the row space, and column space of the matrix $\begin{bmatrix}0&0&3&1&4\\1&3&1&2&1\\3&9&4&5&2\\4&12&8&8&7\end{bmatrix}$.	*1*	*1*	**7**
	c)	The vectors $u_1=(1,2,0)$ and $u_2=(1,3,2)$ and $u_3=(0,1,3)$ form a basis S of $\mathbb{R}^3$. Find the coordinate vector $[v]_S$ of $v=(2,7,-4)$ relative to S.	*1*	*1*	**7**
		UNIT – 2			
3	a)	Find the linear transformation $T:R^2\to R^3$ such that $T(-1,0)=(-1,0,2)$ and $T(2,1)=(1,2,1)$.	*1*	*1*	**6**

	b)	Find the basis of the range space $R(T)$, null space $N(T)$ for the linear transformation $T:\mathbb{R}^5 \to \mathbb{R}^4$ given by $T = \begin{bmatrix} 2 & -1 & 1 & -6 & 8 \\ 1 & -2 & -4 & 3 & -2 \\ -7 & 8 & 10 & 3 & -10 \\ 4 & -5 & -7 & 0 & 4 \end{bmatrix}$ and also verify rank-nullity theorem.	*1*	*1*	**7**
	c)	Let $G:\mathbb{R}^3 \to \mathbb{R}^3$ given by $G(x,\ y,\ z) = (y+z,\ x+z,\ x+y)$. (i) Show that G is invertible. (ii) Find G^{-1}.	*1*	*1*	**7**
		OR			
4	a)	The linear transformation $L:\mathbb{R}^3 \to \mathbb{R}^3$ is given by $L(x,y,z) = (x+z, y+z, x+2y+2z)$. Determine whether the vector $u = (2,-1,3)$ is in the range of L.	*1*	*1*	**6**
	b)	Find the matrix of linear transformation $T:V_2(R) \to V_2(R)$ defined by $T(x,\ y) = (2x+3y,\ 4x-5y)$ with respect to basis $S = \{(1,\ 2),(2,\ 5)\}$.	*1*	*1*	**7**
	c)	Find the basis for the range space $R(T)$ and the kernel $N(T)$ of the linear transformation $T:\mathbb{R}^4 \to \mathbb{R}^3$ given by the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 3 & 8 & 13 & -3 \end{bmatrix}$. Hence verify the rank-nullity theorem.	*1*	*1*	**7**
		UNIT - 3			
5	a)	Apply Cayley-Hamilton theorem to compute A^4 if $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$.	*1*	*1*	**6**
	b)	Find the eigenspace of the linear transformation $T:R^3 \to R^3$ given by $T(x,y,z) = (3x+2y+z\ ,\ x+4y+z,\ x+2y+3z)$.	*1*	*1*	**7**
	c)	Find the characteristic and minimal polynomial of $\begin{bmatrix} 4 & -1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$.	*1*	*1*	**7**
		OR			
6	a)	Apply Cayley-Hamilton theorem to find A^{-1} if $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$.	*1*	*1*	**6**
	b)	Determine the eigenspaces of the linear transformation $T:\mathbb{R}^3 \to \mathbb{R}^3$ defined by $T = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.	*1*	*1*	**7**
	c)	Write all possible Jordan canonical form of the linear transformation $T:\mathbb{R}^3 \to \mathbb{R}^3$ when $\Delta(t) = (t-5)^2 (t-7)^3$ is the characteristic polynomial.	*1*	*1*	**7**

<table>
<tr><td></td><td></td><td></td><td>UNIT – 4</td><td></td><td></td><td></td></tr>
<tr><td></td><td>7</td><td>a)</td><td>Find the angle between the vectors $A=\begin{bmatrix}1 & -1\\ 2 & 3\\ 3 & 4\end{bmatrix}$ and $B=\begin{bmatrix}1 & 2\\ 2 & -2\\ -3 & 5\end{bmatrix}$ where $\langle A,B\rangle = Tr\left(B^T A\right)$.</td><td>1</td><td>1</td><td>4</td></tr>
<tr><td></td><td></td><td>b)</td><td>Obtain the QR decomposition of the matrix $A=\begin{bmatrix}1 & 0 & -1\\ 2 & -3 & 3\\ -1 & 2 & 4\end{bmatrix}$.</td><td>1</td><td>1</td><td>8</td></tr>
<tr><td></td><td></td><td>c)</td><td>A sales organization obtains the following data relating the number of salespersons to annual sales.
<table><tr><td>Number of salespersons</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr><tr><td>Annual Sales (millions of dollars)</td><td>2.3</td><td>3.2</td><td>4.1</td><td>5.0</td><td>6.1</td><td>7.2</td></tr></table>Let x denote the number of salespersons and y denote the annual sales (in millions of dollars). Find the least squares line of the form $y=a+bx$.</td><td>2</td><td>1</td><td>8</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>8</td><td>a)</td><td>If $P_2(t)$ is the vector space of polynomials of degree ≤ 2 with $\langle f,\ g\rangle = \int_0^1 f(t)g(t)dt$. Find a basis of the subspace W orthogonal to $h(t)=2t+1$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Show that $S=\{(1,1,0,-1),(1,2,1,3),(1,1,-9,2),(16,-13,1,3)\}$ is orthogonal basis of $\mathbb{R}^4$. Hence Find the coordinates of the vector $v=(1,1,-1,1)$ in $\mathbb{R}^4$ relative the basis S.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Find an orthogonal matrix P whose first row is $u_1=\left(\frac{1}{3},\frac{2}{3},\frac{2}{3}\right)$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT – 5</td><td></td><td></td><td></td></tr>
<tr><td></td><td>9</td><td>a)</td><td>Determine the modal matrix that reduces the quadratic form $x^2+5y^2+z^2+2xy+6xz+2yz$ to its canonical form and hence discuss the nature the quadratic form.</td><td>2</td><td>1</td><td>10</td></tr>
<tr><td></td><td></td><td>b)</td><td>Reduce the dimension from two to one using principal component analysis for the following data:
<table><tr><td>x</td><td>4</td><td>8</td><td>13</td><td>7</td></tr><tr><td>y</td><td>11</td><td>4</td><td>5</td><td>14</td></tr></table></td><td>2</td><td>1</td><td>10</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>10</td><td>a)</td><td>Orthogonally diagonalize $A=\begin{bmatrix}2 & 2 & 4\\ 2 & 5 & 8\\ 4 & 8 & 17\end{bmatrix}$.</td><td>1</td><td>1</td><td>10</td></tr>
<tr><td></td><td></td><td>b)</td><td>Determine a singular value decomposition of $A=\begin{bmatrix}1 & -1\\ -2 & 2\\ 2 & -2\end{bmatrix}$.</td><td>1</td><td>1</td><td>10</td></tr>
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