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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

June 2025 Semester End Main Examinations

Programme: B.E. **Semester: IV**
Branch: CSE / ISE / AI and ML **Duration: 3 hrs.**
Course Code: 19MA4BSLIA **Max Marks: 100**
Course: Linear Algebra

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - I	*CO*	*PO*	**Marks**
1	a)	Apply LU decomposition method to solve the system of equation $2x+y+4z=12$, $4x+11y-z=33$ and $8x-3y+2z=20$.	1	1	**6**
	b)	Express the polynomial $v=t^2+4t-3$ in $p(t)$ as a linear combination of the polynomials $p_1=t^2-2t+5$, $p_2=2t^2-3t$ and $p_3=t+3$.	1	1	**7**
	c)	Let W be a subspace of $\mathbb{R}^4$ spanned by the vectors $u_1=(1,-2,5,-3)$, $u_2=(2,3,1,-4)$ and $u_3=(3,8,-3,-5)$. (i) Find the basis and dimension of W. (ii) Extend the basis of W to a basis of $\mathbb{R}^4$.	1	1	**7**
		OR			
2	a)	Apply Gauss elimination method to solve the system of equation $20x+y-2z=17$, $3x+20y-z=-18$ and $2x-3y+20z=25$.	1	1	**6**
	b)	Show that the vectors $u_1=(1,1,1)$, $u_2=(1,2,3)$, $u_3=(1,5,8)$ span $\mathbb{R}^3$.	1	1	**7**
	c)	Find the dimension and a basis of the solution space W of the system of equations $x+2y+2z-s+3t=0$; $x+2y+3z+s+t=0$ and $3x+6y+8z+s+5t=0$.	1	1	**7**
		UNIT-II			
3	a)	If $T:\mathbb{R}^3\to\mathbb{R}^3$ defined by $T(x,y,z)=(x-y,y+z,x+z)$, find $N(T)$ and $R(T)$ and verify rank nullity theorem.	1	1	**6**
	b)	Consider T, a linear operator on $\mathbb{R}^3$ defined by $T(x,y,z)=(2x+3y-z,4y-z,2z)$. Is T invertible? If so, then find a formula for T^{-1}.	1	1	**7**
	c)	Find the matrix of linear transformation $T:V_2(\mathbb{R})\to V_3(\mathbb{R})$ defined by $T(x,y)=(-x+2y,y,-3x+3y)$ relative to the bases $B_1=\{(1,1),(-1,1)\}$ and $B_2=\{(1,1,1),(1,-1,1),(0,0,1)\}$.	1	1	**7**
		OR			

	4	a)	Find the matrix representation with respect to basis $\{(1,2),(0,-3)\}$ for R^2 and the basis $S=\{(1,1,0),(0,1,1),(0,1,-1)\}$ in $\mathbb{R}^3$ for the linear transformation $T:\mathbb{R}^2\to\mathbb{R}^3$ defined by $T(a,b)=(4a+b,3a,2a-b)$.	*1*	*1*	**6**
		b)	Determine whether or not the linear mapping $T:\mathbb{R}^3\to\mathbb{R}^3$ defined by $T(x,y,z)=(x+2y+3z,3x+4y+4z,7x+10y+12z)$ is invertible and hence find T^{-1} if invertible.	*1*	*1*	**7**
		c)	Find the bases for the image space and null space and hence verify Rank-Nullity theorem for the linear transformation $T:\mathbb{R}^3\to\mathbb{R}^3$ defined by $T(x,y,z)=(x+2y-3z,\ 2x-4y+7z,\ 2y+z)$.	*1*	*1*	**7**
			UNIT - III			
	5	a)	Find A^4 given $A=\begin{bmatrix}1&2&3\\2&4&5\\3&5&6\end{bmatrix}$ using Cayley-Hamilton theorem.	*1*	*1*	**6**
		b)	Find the eigen space of the linear transformation $T:R^3\to R^3$ defined by $T(x,y,z)=(3x+2y+z,\ x+4y+z,\ 2y+4z)$	*1*	*1*	**7**
		c)	Determine all possible Jordan canonical forms of the linear operator $T:V\to V$ whose characteristic polynomial is $\Delta(t)=(t-2)^3(t-5)^2$.	*1*	*1*	**7**
			OR			
	6	a)	Find the characteristic and minimal polynomial of the matrix $A=\begin{bmatrix}3&2&0&0&0\\1&4&0&0&0\\0&0&3&1&0\\0&0&1&3&0\\0&0&0&0&4\end{bmatrix}$.	*1*	*1*	**6**
		b)	Find eigenvalues and eigenvectors of the linear transformation $T:V_2(\mathbb{R})\to V_2(\mathbb{R})$ defined by $T(x,y)=(2x+5y,\ 4x+3y)$.	*1*	*1*	**7**
		c)	Write the initial-value problem $\frac{d^2x}{dt^2}-2\frac{dx}{dt}-3x(t)=0$, $x(0)=4$, $\frac{dx(0)}{dt}=5$ in its fundamental form and hence solve.	*1*	*1*	**7**
			UNIT - IV			
	7	a)	Find the projection of the vector $v=t+1$ along $w=t^2+3$ in $P(t)$ with respect to $\langle f,\ g\rangle=\int_0^1 f(t)g(t)\,dt$.	*1*	*1*	**6**
		b)	Find an orthogonal basis and hence an orthonormal basis of the subspaces U of $\mathbb{R}^4$ spanned by the vectors $(1,1,1,1)$, $(1,1,2,4)$ and $(1,2,-4,-3)$ using Gram-Schmidt orthogonalization process.	*1*	*1*	**7**

		c)	Find a QR factorization of $A=\begin{bmatrix}1&1&2\\0&0&1\\1&0&0\end{bmatrix}$.	1	1	**7**
			OR			
	8	a)	Find the angle between the vectors $A=\begin{bmatrix}1&-1\\2&3\\3&4\end{bmatrix}$ and $B=\begin{bmatrix}1&2\\2&-2\\-3&5\end{bmatrix}$ where $\langle A,B\rangle=Tr\left(B^T A\right)$.	1	1	**6**
		b)	Let W be a subspace of $\mathbb{R}^5$ spanned by $u=(1,2,3,-1,2)$ and $v=(2,4,7,2,-1)$. Find a basis of the orthogonal complement of W.	1	1	**7**
		c)	Solve the following system of equations $AX=b$ by the method of least squares $A=\begin{bmatrix}2&1\\1&0\\0&-1\\-1&1\end{bmatrix}$ and $b=\begin{bmatrix}3\\1\\2\\-1\end{bmatrix}$.	1	1	**7**
			UNIT-V			
	9	a)	Orthogonally diagonalize $\begin{bmatrix}3&-2&4\\-2&6&2\\4&2&3\end{bmatrix}$.	1	1	**10**
		b)	Find the singular value decomposition of $\begin{bmatrix}1&-1\\-2&2\\2&-2\end{bmatrix}$.	1	1	**10**
			OR			
	10	a)	Find the modal matrix P such that the matrix $A=\begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix}$ is diagonalizable and hence find A^4.	1	1	**10**
		b)	Find the matrix of transformation which reduces the quadratic form $3x^2+3y^2+3z^2+2xy+2xz-2yz$ in to its canonical form. Also write its nature and canonical form.	1	1	**10**

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