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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

June 2025 Semester End Main Examinations

Programme: B.E.
Branch: CS / IS / AI and ML
Course Code: 22MA4BSLIA
Course: Linear Algebra

Semester: IV
Duration: 3 hrs.
Max Marks: 100

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	*CO*	*PO*	**Marks**
1	a)	Determine whether the set $V=\{(x,y)\mid x,y\in\mathbb{R}\}$ is a vector space over the field of reals when the vector addition is the standard vector addition and the scalar multiplication is defined as $k\cdot(x,y)=(0,ky)$.	*1*	*1*	**6**
	b)	Find a basis and dimension of the subspace W of R^4 spanned by the vectors $u_1=(1,-2,5,-3)$, $u_2=(2,3,1,-4)$ and $u_3=(3,8,-3,-5)$. Also extend the basis of W to a basis of R^4.	*1*	*1*	**7**
	c)	Find the basis and the dimension of the solution space of the homogeneous system of equations $x_1+2x_2-2x_3+2x_4-x_5=0$, $x_1+2x_2-x_3+3x_4-2x_5=0$ and $2x_1+4x_2-7x_3+x_4+x_5=0$.	*1*	*1*	**7**
		OR			
2	a)	Determine which of the following are subspaces. (i) $W=\{(x,y,z),\ 2x-3y+z-1=0\}$ in $\mathbb{R}^3$. (ii) $W=\{(x,y,z),\ x+2y-3z=0\}$ in $\mathbb{R}^3$.	*1*	*1*	**6**
	b)	Express the polynomial $v=t^2+4t-3$ in $p(t)$ as a linear combination of the polynomials $p_1=t^2-2t+5$, $p_2=2t^2-3t$ and $p_3=t+3$.	*1*	*1*	**7**
	c)	Find the basis and dimension of the row space and column space of the following matrix $\begin{bmatrix}1&-3&4&-2&5&4\\2&-6&9&-1&8&2\\2&-6&9&1&9&7\\-1&3&-4&2&-5&-4\end{bmatrix}$.	*1*	*1*	**7**
		UNIT - 2			
3	a)	Find a linear transformation $T:R^2\to R^3$ such that $T(-1,0)=(-1,0,2)$ and $T(2,1)=(1,2,1)$.	*1*	*1*	**6**
	b)	Given $T:R^4\to R^3$ where $T=\begin{bmatrix}1&2&3&1\\1&3&5&-2\\3&8&13&-3\end{bmatrix}$, find a basis of the image set and the null set. Also verify rank-nullity theorem.	*1*	*1*	**7**

		c)	Verify whether the linear map $G:R^2 \to R^3$ defined by $G(x,y)=(x+y,\ x-2y,\ 3x+y)$ is non-singular. Find G^{-1}, if it exists. If not, justify.	1	1	**7**
			OR			
	4	a)	Determine whether the vector $u=(2,-1,3)$ is in the range of the linear transformation $L:\mathbb{R}^3 \to \mathbb{R}^3$ given by $L(x,y,z)=(x+z,y+z,x+2y+2z)$.	1	1	**6**
		b)	Consider the matrix $A=\begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$ that defines a linear operator on R^2. Find the matrix of the linear transformation relative to the basis $S=\{u_1, u_2\}=\left\{\begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ -7 \end{bmatrix}\right\}$.	1	1	**7**
		c)	Let $L:P_2(t)\to P_1(t)$ be the linear transformation defined by $L(at^2+bt+c)=(a+2b)t+(b+c)$. i. Is $-4t^2+2t-2$ in kernel of L? ii. Is t^2+2t+1 in range of L? iii. Find the basis of kernel of L. iv. Find the basis and dimension of range of L.	1	1	**7**
			UNIT - 3			
	5	a)	Apply Cayley –Hamilton theorem to find A^4 if $A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$.	1	1	**6**
		b)	Obtain the eigen spaces of the linear transformation $T:R^3 \to R^3$ defined by $T(x,y,z)=(2x+y,\quad y-z,\quad 2y+4z)$.	1	1	**7**
		c)	Find all possible Jordan canonical forms of the linear transformation T, whose characteristic and minimal polynomials are $f(t)=(t+8)^5(t-8)^4$ and $m(t)=(t-8)^2(t+8)^2$ respectively.	1	1	**7**
			OR			
	6	a)	Apply Cayley-Hamilton theorem to find A^{-1} if $A=\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$.	1	1	**6**
		b)	Obtain the eigen space for the linear transformation $T:P_2(t)\to P_2(t)$ defined by $T(at^2+bt+c)=(2a-c)t^2+(2a+b-2c)t+(-a+2c)$.	1	1	**7**
		c)	Find the characteristic and minimal polynomials of the matrix $A=\begin{bmatrix} 3 & 2 & 0 & 0 & 0 \\ 1 & 4 & 0 & 0 & 0 \\ 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$.	1	1	**7**

			UNIT - 4			
	7	a)	Find the projection of the vector $v = t+1$ along $w = t^2+3$ in $P(t)$ with respect to $\langle f,\ g\rangle = \int_0^1 f(t)g(t)dt$.	*1*	*1*	**6**
		b)	Find an orthogonal basis of the subspace W spanned by the following vectors $S=\{1,\ t,\ t^2\}$ of $P_2(t)$ given $\langle f,g\rangle = \int_0^1 f(t)g(t)dt$.	*1*	*1*	**7**
		c)	Find a least-square solution of the system $AX = B$ and also the least square error if $A=\begin{bmatrix}1 & -3\\ 2 & 6\\ 7 & -3\\ 3 & 4\end{bmatrix}$ and $B=\begin{bmatrix}1\\3\\2\\1\end{bmatrix}$.	*1*	*1*	**7**
			OR			
	8	a)	Find the angle between the vectors $A=\begin{bmatrix}-2 & 3 & 1\\ 2 & 3 & 0\end{bmatrix}$ and $B=\begin{bmatrix}0 & -1 & 2\\ 1 & 3 & 0\end{bmatrix}$ where $\langle A,B\rangle = Tr\left(B^T A\right)$.	*1*	*1*	**6**
		b)	Let W be subspace of R^5, spanned by the vectors $u=(1,2,3,-1,2)$ and $v=(2,4,7,2,-1)$. Find a basis of orthogonal complement of W.	*1*	*1*	**7**
		c)	Find a QR factorization of $A=\begin{bmatrix}1 & 1 & 2\\ 0 & 0 & 1\\ 1 & 0 & 0\end{bmatrix}$.	*1*	*1*	**7**
			UNIT - 5			
	9	a)	Compute the Hessian matrix of the function $f(x,y,z) = -9x^2+6xy-2y^2-2xz-2z^2$ at the point $(0,\ 0,\ 0)$.	*1*	*1*	**4**
		b)	Reduce the quadratic form $3x^2+5y^2+3z^2-2xy+2xz-2yz$ to canonical form by finding the transformation matrix and hence discuss its nature.	*1*	*1*	**9**
		c)	Reduce the dimension from two to one using principal component analysis for the data given below. $\begin{array}{\|c\|c\|c\|c\|c\|}\hline X & 4 & 8 & 13 & 7\\ \hline Y & 11 & 4 & 5 & 14\\ \hline\end{array}$	*1*	*1*	**7**
			OR			
	10	a)	Orthogonally diagonalize $A=\begin{bmatrix}6 & -2 & -1\\ -2 & 6 & -1\\ -1 & -1 & 5\end{bmatrix}$.	*1*	*1*	**10**
		b)	Determine a singular value decomposition of $A=\begin{bmatrix}3 & 2\\ 2 & 3\\ 2 & -2\end{bmatrix}$.	*1*	*1*	**10**

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