U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

June 2025 Semester End Main Examinations

Programme: B.E.
Branch: Civil and Mechanical
Course Code: 15MA4GCMAT
Course: Engineering Mathematics-4

Semester: IV
Duration: 3 hrs.
Max Marks: 100

Instructions:
1. Each unit has an internal choice; answer one complete question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

<table>
<tr><th></th><th></th><th>UNIT - 1</th><th>CO</th><th>PO</th><th>Marks</th></tr>
<tr><td>1</td><td>a)</td><td>Approximate the roots of the equation $xsinx + cosx = 0$ near $x = \pi$ by Newton-Raphson method.</td><td>1</td><td>2,3</td><td>6</td></tr>
<tr><td></td><td>b)</td><td>Evaluate $\int_0^1 \frac{x}{(1+x^2)} dx$ by using Weddle's rule considering six equal strips.</td><td>1</td><td>2,3</td><td>7</td></tr>
<tr><td></td><td>c)</td><td>Apply Euler's Modifier method to solve $\frac{dy}{dx} + y + xy^2 = 0$ with $y(0) = 1$ at $x = 0.1$. Perform three modifications.</td><td>1</td><td>2,3</td><td>7</td></tr>
<tr><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td>2</td><td>a)</td><td>Use Lagrange's interpolation formula to find y at $x = 10$, given
<table><tr><td>x</td><td>5</td><td>6</td><td>9</td><td>11</td></tr><tr><td>y</td><td>12</td><td>13</td><td>14</td><td>16</td></tr></table>.</td><td>1</td><td>2,3</td><td>6</td></tr>
<tr><td></td><td>b)</td><td>Apply fourth order Runge-Kutta method to find y at $x = 0.1$ given that $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ and $h = 0.1$.</td><td>1</td><td>2,3</td><td>7</td></tr>
<tr><td></td><td>c)</td><td>Approximate the value of $I = \int_4^{5.2} \log x \, dx$ using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule by considering seven ordinates.</td><td>1</td><td>2,3</td><td>7</td></tr>
<tr><td></td><td></td><td>UNIT – 2</td><td></td><td></td><td></td></tr>
<tr><td>3</td><td>a)</td><td>Derive the Crank-Nicolson implicit formula for the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$.</td><td>2</td><td>2,3</td><td>6</td></tr>
<tr><td></td><td>b)</td><td>Find the numerical solution of the parabolic equation $3u_t = u_{xx}$ when $u(0,t) = 0, u(4,t) = 0$ and $u(x,0) = x(4-x)$ by taking $h = 1$ and $k = 1$. Compute the values of $u(x,t)$ up to $t = 2$.</td><td>2</td><td>2,3</td><td>7</td></tr>
<tr><td></td><td>c)</td><td>Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(0,t) = 0$, $u(4,t) = 0$, $u_t(x,0) = 0$ and $u(x,0) = x(4-x)$ by taking $h = 1$ and $k = 0.5$ up to two-time levels.</td><td>2</td><td>2,3</td><td>7</td></tr>
<tr><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
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	4	a)	Derive the numerical solution of one-dimensional wave equation $u_{tt} = c^2 u_{xx}$.	2	2,3	**6**
		b)	Solve $u_t = u_{xx}$ subject the conditions $u(0,t) = u(1,t) = 0$ and $u(x,0) = \sin \pi x, 0 \le x \le 1, 0 \le t \le 0.1$ by taking $h = 0.2$ and $\alpha = 1/2$.	2	2,3	**7**
		c)	Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(0,t) = u(5,t) = 0$, $u_t(x,0) = 0$ and $u(x,0) = x^2(5-x)$ by taking $h = 1$, $k = 0.25$ up to two-time levels.	2	2,3	**7**
			UNIT – 3			
	5	a)	Obtain the orthogonal trajectories of the family of curves $v(r,\theta) = \left(r - \frac{1}{r}\right)\sin\theta = c$, for $r \ne 0$.	3	2	**6**
		b)	Discuss the transformation $w = z^2$.	3	2	**7**
		c)	Find the bilinear transformation which maps $z = 1,\ i, -1$ onto $w = i,\ 0,\ -i$.	3	2	**7**
			OR			
	6	a)	State and prove Cauchy-Riemann equations in the Cartesian form.	3	2	**6**
		b)	Construct analytic function $f(z) = u + iv$, where $u = e^{2x}(x\cos 2y - y\sin 2y)$.	3	2	**7**
		c)	If $f(z) = u + iv$ is an analytic function of z, then show that $\left[\frac{\partial}{\partial x}\lvert f(z)\rvert\right]^2 + \left[\frac{\partial}{\partial y}\lvert f(z)\rvert\right]^2 = \lvert f'(z)\rvert^2$.	3	2	**7**
			UNIT - 4			
	7	a)	State and prove Cauchy's theorem.	3	2	**6**
		b)	Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent's series valid for the region $1 < \lvert z\rvert < 3$.	3	2	**7**
		c)	Evaluate $\int_c \lvert z\rvert^2\, dz$, where C is a square with the following vertices $(0,0), (1,0), (1,1), (0,1)$.	3	2	**7**
			OR			
	8	a)	Evaluate $\int_C \frac{e^{2z}}{(z+1)(z-2)} dz$ where C is the circle $\lvert z\rvert = 3$, using Cauchy's residue theorem.	3	2	**6**
		b)	Verify Cauchy's theorem for the function $f(z) = z^2$ where C is the square having vertices $(0,0), (1,0), (1,1), (0,1)$.	3	2	**7**
		c)	Evaluate $\int_C \frac{z^2 - z + 1}{z - 1}$, where C is the circle $(i)\ \lvert z\rvert = 1 \quad (ii)\ \lvert z\rvert = \frac{1}{2}$.	3	2	**7**

			UNIT - 5			
	9	a)	Fit a straight line $y = a\,x + b$ to the data given below: x: 5, 10, 15, 20, 25 y: 16, 19, 23, 26, 30	*5*	*3*	**6**
		b)	Determine the coefficient of correlation between industrial production and export from the given data: Production: 55, 56, 58, 59, 60, 60, 62 Export: 35, 38, 38, 39, 44, 43, 45	*5*	*3*	**7**
		c)	Derive the expression for the mean and variance of the Poisson distribution.	*6*	*3*	**7**
			OR			
	10	a)	Fit a second degree parabola of the form $y = a + bx + cx^2$ by the least squares principle for the data given below: x: 0, 1, 2, 3, 4 y: 1, 1.8, 1.3, 2.5, 6.3	*5*	*3*	**6**
		b)	Determine the co-efficient of correlation between x and y given $2\sigma_x = \sigma_y$ and the angle between the lines of regression is $\tan^{-1}\left(\frac{3}{5}\right)$.	*5*	*3*	**7**
		c)	The mean weight of 500 students during a medical exam was found to be 50 kgs and the standard deviation is 6kgs. Assuming that the weights are normally distributed find the number of students having weight i) Between 40 and 50 kgs ii) More than 60kgs.	*6*	*3*	**7**

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