

# B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

## February 2025 Semester End Main Examinations

**Programme: B.E.**

**Branch: Medical Electronics**

**Course Code: 23MA4BSMMD**

**Course: Mathematical Methods for Medical Electronics**

**Semester: IV**

**Duration: 3 hrs.**

**Max Marks: 100**

**Instructions:**

1. All units have internal choice, answer one complete question from each unit.
2. Missing data, if any, may be suitably assumed.
3. Use of statistical tables is permitted.

**Important Note:** Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1		CO	PO	Marks																	
1	a)	If $f(z)$ is an analytic function of $z$ , then prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)( f(z) ^2) = 4 f'(z) ^2$ .		1	1	<b>6</b>																	
	b)	Determine the imaginary part of the complex function $f(z)$ whose real part is $e^{2x}(x\cos(2y) - y\sin(2y))$ .		1	1	<b>7</b>																	
	c)	State and prove Cauchy's theorem.		1	1	<b>7</b>																	
		<b>OR</b>																					
2	a)	Find the analytic function $f(z)$ where $v = r^2 \cos(2\theta) - r \cos(\theta) + 2$ using Milne-Thompson method.		1	1	<b>6</b>																	
	b)	Discuss the transformation $w = z^2$ .		1	1	<b>7</b>																	
	c)	Apply Cauchy's integral formula to evaluate $\int_C \frac{e^z}{(z^2 + \pi^2)^2} dz$ over the circle $C:  z  = 4$ .		1	1	<b>7</b>																	
		<b>UNIT - 2</b>																					
3	a)	The results of measurement of electric resistance $R$ of a copper bar at various temperature $t^0c$ are listed below:		1	1	<b>6</b>																	
		<table border="1"> <tr> <td><math>t</math></td><td>19</td><td>25</td><td>30</td><td>36</td><td>40</td><td>45</td><td>50</td></tr> <tr> <td><math>R</math></td><td>76</td><td>77</td><td>79</td><td>80</td><td>82</td><td>83</td><td>85</td></tr> </table> <p>Find a least squares straight line of the form <math>R = a + bt</math> where <math>a</math> and <math>b</math> are constants.</p>	$t$	19	25	30	36	40	45	50	$R$	76	77	79	80	82	83	85					
$t$	19	25	30	36	40	45	50																
$R$	76	77	79	80	82	83	85																
	b)	The following table shows the recorded data of the test scores made by the salesman on an intelligence test and their weekly sales.		1	1	<b>7</b>																	
		<table border="1"> <tr> <td>Salesman</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr> <tr> <td>Test scores</td><td>92</td><td>89</td><td>87</td><td>86</td><td>83</td></tr> <tr> <td>Sales</td><td>86</td><td>88</td><td>91</td><td>77</td><td>68</td></tr> </table> <p>Calculate the coefficient of correlation between the test scores and the sales. Hence find the regression line of sales on test scores and estimate</p>	Salesman	1	2	3	4	5	Test scores	92	89	87	86	83	Sales	86	88	91	77	68			
Salesman	1	2	3	4	5																		
Test scores	92	89	87	86	83																		
Sales	86	88	91	77	68																		

		the most probable weekly sales volume if a salesman makes a score of 85.																									
	c)	Compute the rank-correlation coefficient for the given data: <table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td><math>x</math></td><td>21</td><td>23</td><td>30</td><td>54</td><td>57</td><td>58</td><td>72</td><td>78</td><td>87</td><td>90</td></tr> <tr> <td><math>y</math></td><td>60</td><td>71</td><td>72</td><td>83</td><td>110</td><td>84</td><td>100</td><td>92</td><td>113</td><td>135</td></tr> </table>	$x$	21	23	30	54	57	58	72	78	87	90	$y$	60	71	72	83	110	84	100	92	113	135	1	1	7
$x$	21	23	30	54	57	58	72	78	87	90																	
$y$	60	71	72	83	110	84	100	92	113	135																	
<b>OR</b>																											
4	a)	The equations of the regression lines of $x$ on $y$ and $y$ on $x$ are $x + 0.87y = 19.13$ and $y + 0.5x = 11.64$ respectively. Find the correlation coefficient and the means of $x$ and $y$ .	1	1	6																						
	b)	If the coefficient of correlation between two variables $x$ and $y$ is 0.5 and the acute angle between their lines of regression is $\tan^{-1}\left(\frac{3}{5}\right)$ , show that $\sigma_x = \frac{1}{2}\sigma_y$ .	1	1	7																						
	c)	Estimate the chlorine residual in a swimming pool 5 hours after it has been treated with chemicals by fitting a curve of the form $y = ax^b$ to the following data: <table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td><math>x</math> (no. of hours)</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td></tr> <tr> <td><math>y</math> (chlorine residual parts/million)</td><td>1.8</td><td>1.5</td><td>1.4</td><td>1.1</td><td>1.1</td><td>0.9</td></tr> </table>	$x$ (no. of hours)	2	4	6	8	10	12	$y$ (chlorine residual parts/million)	1.8	1.5	1.4	1.1	1.1	0.9	1	1	7								
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<b>UNIT – 3</b>																											
5	a)	The joint probability distribution of two random variables $X$ and $Y$ is given below: <table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td></td> <td><math>Y \backslash X</math></td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>2</td> <td></td> <td><math>\frac{1}{6}</math></td> <td><math>\frac{1}{6}</math></td> <td><math>\frac{1}{6}</math></td> </tr> <tr> <td>5</td> <td></td> <td><math>\frac{1}{12}</math></td> <td><math>\frac{1}{12}</math></td> <td><math>\frac{1}{12}</math></td> </tr> <tr> <td>7</td> <td></td> <td><math>\frac{1}{12}</math></td> <td><math>\frac{1}{12}</math></td> <td><math>\frac{1}{12}</math></td> </tr> </table> i) Find the marginal distributions of $X$ and $Y$ ii) $Cov(X, Y)$ .		$Y \backslash X$	3	4	5	2		$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	5		$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	7		$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	1	1	6		
	$Y \backslash X$	3	4	5																							
2		$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$																							
5		$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$																							
7		$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$																							
	b)	A certain screw making machine has a chance of producing 2 defectives out of 1000. The screws are packed in boxes of 100. Apply Poisson distribution to find the approximate number of boxes containing: (i) no defective screw (ii) one defective screw (iii) two defective screws, in a consignment of 5000 boxes.	1	1	7																						
	c)	A manufacturer of air-mail envelopes knows from experience that weight of the envelopes is normally distributed with mean 1.95g and standard deviation 0.05g. About how many envelopes weighing: (i) 2g or more (ii) 2.05g or more can be expected in a given packet of 100 envelopes.	1	1	7																						
<b>OR</b>																											
6	a)	Derive an expression for mean and variance of Poisson distribution.	1	1	6																						
	b)	In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and the standard deviation of the distribution.	1	1	7																						

	c)	The joint probability function of two random variables $X$ and $Y$ is given by $f(x, y) = c(2x + y)$ where $x$ and $y$ can assume all integral values such that $0 \leq x \leq 2, 0 \leq y \leq 3$ and $f(x, y) = 0$ otherwise. Find i) the value of the constant $c$ ii) Marginal Probability distributions of $X$ and $Y$ .	1	1	7																											
		<b>UNIT – 4</b>																														
7	a)	Mice with an average lifespan of 32 months will live up to 40 months when fed by a certain nutritious food. If 64 mice fed on this diet have an average lifespan of 38 months and standard deviation of 5.8 months, is there any reason to believe that the average lifespan is less than 40 months? Use 5% level of significance.	1	1	6																											
	b)	Can we conclude that chronically ill children tend, on the average, to be less self-confident than healthy children? A test designed to measure self-confidence was administered to 16 chronically ill and 21 healthy children. The mean scores and standard deviations were <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <th></th> <th>mean</th> <th>standard deviation</th> </tr> <tr> <td>ill Group</td> <td>22.5</td> <td>4.1</td> </tr> <tr> <td>well Group</td> <td>26.9</td> <td>3.2</td> </tr> </table> Use 1% level of significance.		mean	standard deviation	ill Group	22.5	4.1	well Group	26.9	3.2	1	1	7																		
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ill Group	22.5	4.1																														
well Group	26.9	3.2																														
	c)	Following data give weight gains of 8 pairs of experimental animals matched with respect to various growth factors. One of each pair, selected by chance, was given vitamin $B_{12}$ supplement and the other was not given the supplement. Test whether $B_{12}$ supplement is beneficial at 5% level of significance. <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <th>Pair</th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> <th>6</th> <th>7</th> <th>8</th> </tr> <tr> <td>With <math>B_{12}</math></td> <td>1.6</td> <td>1.68</td> <td>1.75</td> <td>1.64</td> <td>1.75</td> <td>1.79</td> <td>1.78</td> <td>1.77</td> </tr> <tr> <td>Without <math>B_{12}</math></td> <td>1.56</td> <td>1.52</td> <td>1.52</td> <td>1.49</td> <td>1.59</td> <td>1.56</td> <td>1.6</td> <td>1.56</td> </tr> </table>	Pair	1	2	3	4	5	6	7	8	With $B_{12}$	1.6	1.68	1.75	1.64	1.75	1.79	1.78	1.77	Without $B_{12}$	1.56	1.52	1.52	1.49	1.59	1.56	1.6	1.56	1	1	7
Pair	1	2	3	4	5	6	7	8																								
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Without $B_{12}$	1.56	1.52	1.52	1.49	1.59	1.56	1.6	1.56																								
		<b>OR</b>																														
8	a)	An ambulance service company claims that on an average it takes 20 minutes between a call for an ambulance and the patient's arrival at the hospital. If in 6 calls the time taken (between a call and arrival at hospital) are 27, 18, 26, 15, 20 and 32. Can the company's claim be accepted at 1% level of significance?	1	1	6																											
	b)	A random sample of 40 'geysers' produced by company A have a mean lifetime (mlt) of 647 hours of continuous use with a standard deviation of 27 hours, while a sample 40 produced by another company B have mean lifetime (mlt) of 638 hours with standard deviation 31 hours. Does this substantiate the claim of company A that their 'geysers' are superior to those produced by company B at 0.05 level of significance?	1	1	7																											
	c)	Marks obtained in mathematics by 6 students before and after intensive coaching are given below: <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>Before</td> <td>24</td> <td>17</td> <td>18</td> <td>20</td> <td>19</td> <td>23</td> </tr> <tr> <td>After</td> <td>24</td> <td>20</td> <td>22</td> <td>20</td> <td>17</td> <td>24</td> </tr> </table> Test at 5% level of significance whether the intensive coaching is useful?	Before	24	17	18	20	19	23	After	24	20	22	20	17	24	1	1	7													
Before	24	17	18	20	19	23																										
After	24	20	22	20	17	24																										

<b>UNIT - 5</b>																				
9	a)	A commonly prescribed drug for relieving nervous tension is believed to be only 60% effective. Experimental results with a new drug administered to a random sample of 100 adults who were suffering from nervous tension show that 70 received relief. Is this sufficient evidence to conclude that the new drug is superior to the one commonly prescribed? Use $\alpha=5\%$ level of significance.	<i>1</i>	<i>1</i>	<b>6</b>															
	b)	The number of cars passing a given point in 100 five second intervals was observed as follows:	<i>1</i>	<i>1</i>	<b>7</b>															
		<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>No. of Cars</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr> <tr> <td>No. of intervals</td><td>40</td><td>23</td><td>15</td><td>5</td><td>7</td><td>10</td></tr> </table> <p>Fit a Poisson distribution and test for its goodness of fit. Use 5% level of significance.</p>	No. of Cars	0	1	2	3	4	5	No. of intervals	40	23	15	5	7	10				
No. of Cars	0	1	2	3	4	5														
No. of intervals	40	23	15	5	7	10														
10	c)	<p>Two random samples from two normal populations are given below:</p> <p>Sample I: 16    26    27    23    24    22  Sample II: 33    42    35    32    28    31</p> <p>At 1% level of significance, do the estimate of population variances differ significantly?</p>	<i>1</i>	<i>1</i>	<b>7</b>															
		<b>OR</b>																		
	a)	A sample analysis of examination results of 500 students was made. It was found that 220 students had failed, 170 had secured third class, 90 had secured second class and 20 had secured first class. Do these figures support the general examination result which is in the ratio 4:3:2:1 for the respective categories? Use 0.05 level of significance.	<i>1</i>	<i>1</i>	<b>6</b>															
	b)	In a study of nutrition cares in nursing homes, it is found that among 55 patients with hypertension, 24 were on sodium restricted diets. Of 149 patients without hypertension, 36 were on sodium restricted diets. Can we conclude that in the sampled populations, the proportion of patients on sodium restricted diets is higher amongst the patients with hypertension than among the patients without hypertension? Use 0.01 level of significance.	<i>1</i>	<i>1</i>	<b>7</b>															
	c)	Can we conclude that the two population variances are equal for the following data of post graduates passed out from a state and private university.	<i>1</i>	<i>1</i>	<b>7</b>															
		<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>State:</td><td>8350</td><td>8260</td><td>8130</td><td>8340</td><td>8070</td><td>----</td></tr> <tr> <td>Private:</td><td>7890</td><td>8140</td><td>7900</td><td>7950</td><td>7840</td><td>7920</td></tr> </table>	State:	8350	8260	8130	8340	8070	----	Private:	7890	8140	7900	7950	7840	7920				
State:	8350	8260	8130	8340	8070	----														
Private:	7890	8140	7900	7950	7840	7920														

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