U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

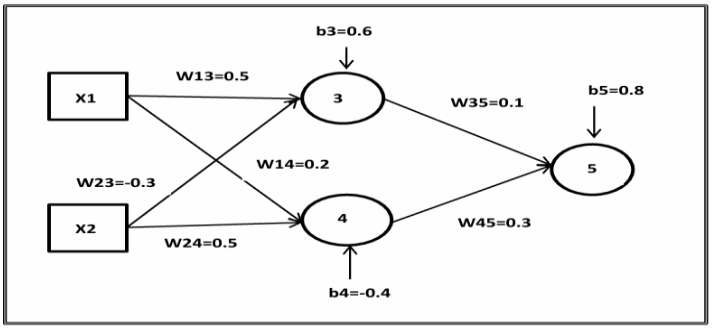
October 2024 Supplementary Examinations

Programme: B.E. **Semester: IV**
Branch: AI and ML **Duration: 3 hrs.**
Course Code: 23MA4BSMML **Max Marks: 100**
Course: Mathematical Foundation for Machine Learning – 2

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	*CO*	*PO*	**Marks**
1	a)	Find pseudo inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$.	1	1	**4**
	b)	Solve the equations $2x + 3y - z = 5,\ 3x + 2y + z = 10$, $x - 5y + 3z = 0$ using Doolittle's decomposition method.	1	1	**8**
	c)	Find the singular value decomposition of the matrix $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$.	1	1	**8**
		OR			
2	a)	Apply Crout's decomposition method to solve the system of equations $x + y + z = 3,\ 2x - y - z = 3,\ x - y + z = 9$.	1	1	**8**
	b)	Apply principal component analysis to project the given 2-dimensional data on to 1-dimension and hence write the first principal component. x: 2, 3, 4, 5, 6, 7 y: 1, 5, 3, 6, 7, 8	1	1	**8**
	c)	Determine the Cholesky factorization of given matrix A, such that $A = \begin{bmatrix} 25 & 15i \\ -15i & 75 \end{bmatrix}$.	1	1	**4**
		UNIT – 2			
3	a)	Write pseudocode to find the gradient of a vector valued function and hence find the gradient of $f = \begin{bmatrix} x^2 + yz \\ e^x + \cos(y) \\ z^3 - xy \end{bmatrix}$.	1	1	**6**
	b)	Find gradient of $A(X) = \begin{bmatrix} x^2 y & xz \\ \sin(wz) & e^y + \cos(xw) \end{bmatrix}$ with respect to a matrix $X = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$.	1	1	**6**
	c)	Suppose $f(x, y, z) = \ln(1 + x^2 + y^3 + z)$, find the multivariate	1	1	**8**

<table>
<tr><td></td><td></td><td></td><td>series expansion up to quadratic order around the point $(1,\ 2,\ -1)$.</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 3</td><td></td><td></td><td></td></tr>
<tr><td></td><td>4</td><td>a)</td><td>Apply automatic differentiation to compute the derivative of the function $f(x) = e^{3x} + \sin(2x)$.</td><td>1</td><td>1</td><td>4</td></tr>
<tr><td></td><td></td><td>b)</td><td>Consider a multilayer feed-forward neural network given below. Let the learning rate be 0.5. Train the network for the training tuples$(x_1 = 1, x_2 = 1, \text{output} = 0)$ and show weights and bias updates by using back-propagation algorithm. Assume that sigmoid activation function is used in the network.

</td><td>1</td><td>1</td><td>8</td></tr>
<tr><td></td><td></td><td>c)</td><td>We have recorded the weekly average price y of a stock over 6 consecutive days and x shows the number of days. Fit the relation of the form $y = a + bx$
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>y</td><td>10</td><td>14</td><td>18</td><td>22</td><td>25</td><td>33</td></tr>
</table>
The initial values are $a = 4.9$ and $b = 4.401$. The learning rate is mentioned as 0.05. Carry out one iteration.</td><td>1</td><td>1</td><td>8</td></tr>
<tr><td></td><td></td><td></td><td>UNIT – 4</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Show that $S = \{(x_1, x_2) : x_1^2 + x_2^2 \le 4\}$ is a convex set.</td><td>1</td><td>1</td><td>5</td></tr>
<tr><td></td><td></td><td>b)</td><td>Given $f(x, y, z) = -9x^2 + 6xy - 2y^2 - 2xz - 2z^2$,
i) find all the stationary points of the function,
ii) find the Hessian matrix and
iii) classify the stationary points and find its extreme value.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Apply gradient descent method to find the minimum of the function $f(x, y) = x - y + 2x^2 + 2xy + y^2$ with an initial guess $X = (0,\ 0)$. Carry out 3 iterations. Also write its pseudocode.</td><td>1</td><td>1</td><td>8</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>6</td><td>a)</td><td>Verify Legendre transformation condition $f(x) = f^{**}(x)$ for the function $f(x) = e^x$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Apply Newton's method to minimize the function $f\ (x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ by taking the starting point as $X_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Let $V_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, V_2 = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, V_3 = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$. Find an implicit description $[f : d]$ of the plane H_1 that passes through V_1, V_2 and V_3.</td><td>1</td><td>1</td><td>7</td></tr>
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			UNIT - 5			
	7	a)	Apply Lagrange's multiplier method to find optimum distances of the point $(3,\ 4,\ 12)$ from the sphere $x^2+y^2+z^2=1$.	*1*	*1*	**6**
		b)	Apply Fibonacci method to minimize the function $f(x)=x^2+\frac{54}{x}$ in the range $(0,\ 5)$ by taking $n=3$.	*1*	*1*	**7**
		c)	Derive the Karush-Kuhn-Tucker conditions to find the minimum value of function $Z=12x_1+21x_2+2x_1x_2-2x_1^2-2x_2^2$ subject to the constraints $x_1+x_2\le 10$, $x_2\le 8$ and $x_1, x_2\ge 0$.	*1*	*1*	**7**

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