U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

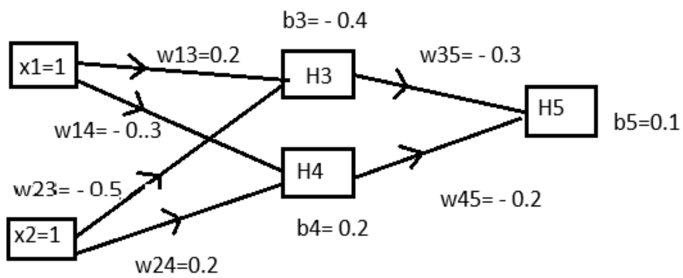
June 2025 Semester End Main Examinations

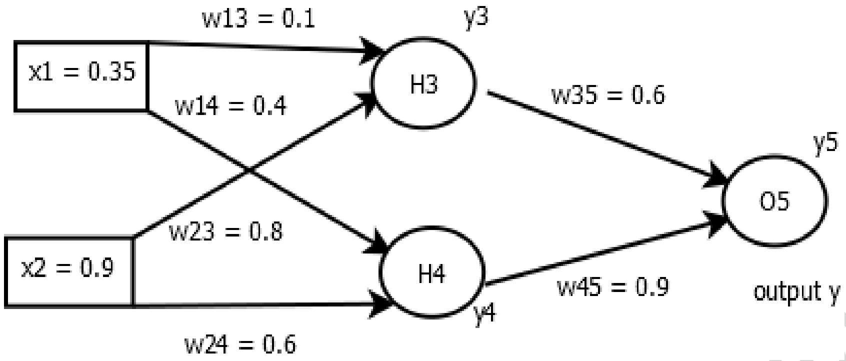
Programme: B.E. **Semester: IV**
Branch: Artificial Intelligence and Machine Learning **Duration: 3 hrs.**
Course Code: 23MA4BSMML **Max Marks: 100**
Course: Mathematical Foundation for Machine Learning – 2

Instructions:
1. Each unit has an internal choice; answer one complete question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	***CO***	***PO***	**Marks**
1	a)	Compute the singular value decomposition of $X=\begin{bmatrix}1 & -1\\ 0 & 1\\ 1 & 0\end{bmatrix}$.	*1*	*1*	**8**
	b)	Apply Crout's method to find the inverse of the matrix $A=\begin{bmatrix}2 & 5 & 2\\ 3 & -2 & 4\\ -6 & 1 & -7\end{bmatrix}$.	*1*	*1*	**7**
	c)	Find the Moore-Penrose pseudoinverse of the matrix $A=\begin{bmatrix}4 & 0\\ 3 & -5\end{bmatrix}$.	*1*	*1*	**5**
		OR			
2	a)	Find the inverse of the matrix $A=\begin{bmatrix}4 & -2 & -6\\ -2 & 10 & 9\\ -6 & 9 & 14\end{bmatrix}$ using Cholesky's decomposition method.	*1*	*1*	**6**
	b)	Apply Doolittle's method to solve the system of equations $x+2y-z=-5;\ 2x+7y-8z=-19;\ -x-8y+15z=25$.	*1*	*1*	**7**
	c)	Find the Principal component for the data $(1,3),(2,2)$ and $(3,1)$.	*1*	*1*	**7**
		UNIT - 2			
3	a)	Verify that $\frac{\partial(a^TX)}{\partial X}=a^T$, given $X=\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}$ and $a=\begin{bmatrix}1\\ 2\\ 3\\ 1\end{bmatrix}$.	*1*	*1*	**5**
	b)	Find the multivariate series expansion of the function $\log\left(\sqrt{1+x+y}\right)$ up to quadratic order about the point $(0,1)$.	*1*	*1*	**8**

<table>
<tr><td></td><td></td><td>c)</td><td>Write the pseudocode to find the gradient of a matrix with respect to a matrix and hence find the gradient of $\begin{bmatrix} {x_0}^2 + x_1x_2 & {x_1}^2x_2 - x_3 \\ \frac{\left(x_1{x_2}^2\right)}{x_3} & x_1{x_2}^3 \end{bmatrix}$ with respect to the matrix $X = \begin{bmatrix} x_0 & x_1 \\ x_2 & x_3 \end{bmatrix}$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>4</td><td>a)</td><td>Write a pseudocode to find the gradient of a vector with respect to a matrix, and hence find the gradient of $f = [e^{x_0x_1} \quad e^{x_2x_3}]$ with respect to $X = \begin{bmatrix} x_0 & x_1 \\ x_2 & x_3 \end{bmatrix}$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>For the given matrix $f = \begin{bmatrix} {x_0}^2 + x_1x_2 & {x_1}^2x_2 - x_3 \\ \frac{\left(x_1{x_2}^2\right)}{x_3} & x_1{x_2}^3 \end{bmatrix}$ and $X = \begin{bmatrix} x_0 & x_1 \\ x_2 & x_3 \end{bmatrix}$, verify that $\frac{\partial(\text{Trace} f(X))}{\partial X} = \text{Trace}\left(\frac{\partial f(X)}{\partial X}\right)$.</td><td>1</td><td>1</td><td>8</td></tr>
<tr><td></td><td></td><td>c)</td><td>Create a linear model of the function $f(x, y, z, u) = x^2y + e^zx + logu$ about $(0, 0, 1, 2)$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 3</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Construct a computation graph of the function $f(x) = \log\left(\sin(x^3)\right) + \sqrt{1 + \cos(x^3)}$. Also find $\frac{df}{dx}$ using automatic differentiation.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Assume that the neuron has a sigmoid activation function, perform a forward pass and a backward pass on the network. Assume that the actual output of y is 1 and learning rate is 0.9.

</td><td>1</td><td>1</td><td>8</td></tr>
<tr><td></td><td></td><td>c)</td><td>For the data given below, fit a linear regression line $y = a + bx$ using the gradient descent method.
<table><tr><td>x</td><td>2</td><td>3</td><td>6</td><td>8</td></tr><tr><td>y</td><td>3</td><td>7</td><td>5</td><td>10</td></tr></table>Initialize the weights $a = 1.5$ and $b = 0.95$. Update the weights such that the error is minimum taking learning rate as 0.1. Perform one iteration.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
</table>

	6	a)	Construct a computation graph of the function $f(x\,,y)=\ln(x)+xy-siny$ and hence compute the derivative of f with respect to x and y at the point $(2,5)$.	1	1	**7**
		b)	Assume that the neuron has a sigmoid activation function, perform a forward pass and a backward pass on the network. Assume that the actual output of y is 0.5 and learning rate is 1. 	1	1	**7**
		c)	Consider the data given below and fit a linear regression line $y=ax+b$ using gradient descent method. x: -3, -2, 1, 2, 3 y: 5, 7, 9, 11, 13 Initialize the weights a and b as 0 and learning rate 0.001. Update the weights so that the error is minimal. Perform one iteration.	1	1	**6**
			UNIT - 4			
	7	a)	Check whether the set $S=\{(x_1,x_2,x_3): x_1^2+x_2^2+x_3^2\le 1\}$ is a convex set or not.	1	1	**6**
		b)	Find the local optimum for the function $f(x_1,x_2)=x_1^2+x_2^2+x_1x_2+10(x_1+x_2)$ using Gradient descent method starting at $\begin{bmatrix}1\\1\end{bmatrix}$. Perform two iterations.	1	1	**7**
		c)	Let $A=\left\{\begin{bmatrix}2\\-1\\5\end{bmatrix},\begin{bmatrix}3\\1\\3\end{bmatrix},\begin{bmatrix}-1\\6\\0\end{bmatrix}\right\}$, $B=\left\{\begin{bmatrix}0\\5\\-1\end{bmatrix},\begin{bmatrix}1\\-3\\-2\end{bmatrix},\begin{bmatrix}2\\2\\1\end{bmatrix}\right\}$ and $n=\begin{bmatrix}3\\1\\-2\end{bmatrix}$, find a hyperplane H with normal n that separates A and B.	1	1	**7**
			OR			
	8	a)	Prove that $f^{**}(x)=f(x)$, where $f(x)=cx^2,\ c>0$.	1	1	**5**
		b)	Find the local optimum using the Hessian matrix method for the function $f(x,y,z)=-x^3+3xz+2y-y^2-3z^2$.	1	1	**8**
		c)	Find the local optimum for the function $f(x,y)=x-y+2x^2+2xy+2y^2$ near $(0,0)$ using Newton's method. Perform two iterations.	1	1	**7**
			UNIT - 5			
	9	a)	Apply the three-point interval search method to find the maximum of $f(x)=x\sin(4x)$ on $[0,3]$. Perform three iterations.	1	1	**6**

		b)	Maximize the function $z = 12x_1 + 21x_2 + 2x_1x_2 - 2x_1^2 - 2x_2^2$ subject to the constraints $x_1 + x_2 \le 10$; $x_2 \le 8$; $x_1, x_2 \ge 0$ by deriving the Karush-Kuhn-Tucker conditions.	*1*	*1*	**7**
		c)	Apply Lagrange's multiplier method to find the maximum and minimum values of $f(x, y, z) = 4y - 3z$ subject to the constraints $2x - y - z = 2$ and $x^2 + y^2 = 1$.	*1*	*1*	**7**
			OR			
	10	a)	Apply the Fibonacci method to minimize the function $f(x) = x(x - 1.5)$ in the range $[0,1]$ by taking $n = 4$.	*1*	*1*	**6**
		b)	Maximize the function $z = (x_1 - 2)^2 + (x_2 - 1)^2$ subject to the constraints $x_1^2 - x_2 \le 0; x_1 + x_2 \le 2,\ x_1, x_2 > 0$ by deriving the Karush-Kuhn-Tucker conditions.	*1*	*1*	**7**
		c)	Apply Lagrange's multiplier method to find the maximum and minimum values of $f(x, y) = 5x - 3y$ subject to the constraint $x^2 + y^2 = 136$.	*1*	*1*	**7**

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