

U.S.N.

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

December 2023 Supplementary Examinations

Programme: B.E.

Branch: Chemical Engineering

Course Code: 22MA4BSSAP

Course: Statistics & Probability

Semester: IV

Duration: 3 hrs.

Max Marks: 100

- Instructions:**
1. Answer any FIVE full questions, choosing one full question from each unit.
 2. Missing data, if any, may be suitably assumed.
 3. Use of Statistical tables is permitted.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - I	CO	PO	Marks																		
1	a)	<p>The results of measurement of electric resistance R of a copper bar at various temperature $t^{\circ}C$ are listed below:</p> <table><tr><td>t</td><td>19</td><td>25</td><td>30</td><td>36</td><td>40</td><td>45</td><td>50</td></tr><tr><td>R</td><td>76</td><td>77</td><td>79</td><td>80</td><td>82</td><td>83</td><td>85</td></tr></table> <p>Find a least squares straight line of the form $R = a + bt$ where a and b are constants.</p>	t	19	25	30	36	40	45	50	R	76	77	79	80	82	83	85	CO1	PO1	6		
t	19	25	30	36	40	45	50																
R	76	77	79	80	82	83	85																
	b)	<p>The following table shows the recorded data of the test scores made by the salesman on an intelligence test and their weekly sales.</p> <table><tr><td>Salesman</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>Test scores</td><td>92</td><td>89</td><td>87</td><td>86</td><td>83</td></tr><tr><td>Sales (000)</td><td>86</td><td>88</td><td>91</td><td>77</td><td>68</td></tr></table> <p>Calculate the coefficient of correlation between the test scores and the sales. Hence find the regression line of sales on test scores and estimate the most probable weekly sales volume if a salesman makes a score of 85.</p>	Salesman	1	2	3	4	5	Test scores	92	89	87	86	83	Sales (000)	86	88	91	77	68	CO1	PO1	7
Salesman	1	2	3	4	5																		
Test scores	92	89	87	86	83																		
Sales (000)	86	88	91	77	68																		
	c)	<p>In a certain examination, the percentage of candidates passing and getting distinctions were 45 and 9 respectively. Estimate the average marks and standard deviation obtained by the candidates, the minimum pass and distinction marks being 40 and 75 respectively. Assume that marks are normally distributed.</p>	CO1	PO1	7																		
		OR																					
2	a)	<p>Estimate the chlorine residual in a swimming pool 5 hours after it has been treated with chemicals by fitting an exponential curve of the form $y = ab^x$ to the following data</p> <table><tr><td>x (no. of hours)</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td></tr><tr><td>y (chlorine residual parts/million)</td><td>1.8</td><td>1.5</td><td>1.4</td><td>1.1</td><td>1.1</td><td>0.9</td></tr></table>	x (no. of hours)	2	4	6	8	10	12	y (chlorine residual parts/million)	1.8	1.5	1.4	1.1	1.1	0.9	CO1	PO1	6				
x (no. of hours)	2	4	6	8	10	12																	
y (chlorine residual parts/million)	1.8	1.5	1.4	1.1	1.1	0.9																	

	b)	2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses has (i) No defective fuses (ii) at least 3 defective fuses (iii) at most 3 defective fuses.	CO1	PO1	7												
	c)	i) If θ is an acute angle between the two regression lines, then show that $\tan \theta = \frac{1-r^2}{r} \left(\frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2} \right)$. ii) If the coefficient of correlation between two variables x and y is 0.5 and the acute angle between their lines of regression is $\tan^{-1}\left(\frac{3}{5}\right)$, find the ratio of the standard deviations of x and y.	CO1	PO1	7												
		UNIT – II															
3	a)	Define Stochastic matrix and fixed probability vector. Find the fixed probability vector of $P = \begin{bmatrix} 0 & 3/4 & 1/4 \\ 1/2 & 1/2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.	CO1	PO1	6												
	b)	Two flashcards are selected at random from a box which contains five cards numbered 1,2,2,3,3. Find the joint distributions of X and Y where X denotes the sum of two numbers and Y denote the minimum of two numbers drawn. Also determine $Cov(X, Y)$.	CO1	PO1	7												
	c)	In a certain city, the weather on a day is reported as sunny, cloudy or rainy. If a day is sunny, the probability that the next day is sunny is 70%, cloudy is 20% and rainy is 10%. If a day is cloudy, the probability that the next day is sunny is 30%, cloudy is 20% and rainy is 50%. If a day is rainy, the probability that the next day is sunny is 30%, cloudy is 30% and rainy is 40%. If a Sunday is sunny, find the probability that the Wednesday is rainy.	CO1	PO1	7												
		UNIT – III															
4	a)	In a true-false test, a test item is considered to be good if it discriminates between well-prepared students and poorly prepared students. If 205 of 250 well- prepared students and 137 of 250 poorly prepared students answer a certain item correctly, test at 0.01 level of significance whether for the given item the proportion of correct answers can be expected to be higher among well-prepared students than among poorly prepared students.	CO2	PO1	6												
	b)	The following table presents the data on the values of a harvested crop stored in the open and inside a granary. <table border="1"><thead><tr><th></th><th>Sample sizes</th><th>Mean</th><th>$\sum_i (x_i - \bar{x})^2$</th></tr></thead><tbody><tr><td>Outside</td><td>40</td><td>117</td><td>8685</td></tr><tr><td>Inside</td><td>100</td><td>132</td><td>27315</td></tr></tbody></table> Assuming that the 2 samples are random, examine if the mean value of the harvested crop is affected by the weather conditions. (Use $\alpha=1\%$).		Sample sizes	Mean	$\sum_i (x_i - \bar{x})^2$	Outside	40	117	8685	Inside	100	132	27315	CO2	PO1	7
	Sample sizes	Mean	$\sum_i (x_i - \bar{x})^2$														
Outside	40	117	8685														
Inside	100	132	27315														

	c)	The mean yield of two sets of plots and their variability are as given below. Examine whether the difference in the variability in yields is significant at 1% level of significance. <table><tr><td></td><td>Set of 121 plots</td><td>Set of 81 plots</td></tr><tr><td>Mean yield per plot</td><td>84 lb</td><td>81 lb</td></tr><tr><td>Standard deviation per plot</td><td>10</td><td>12</td></tr></table>		Set of 121 plots	Set of 81 plots	Mean yield per plot	84 lb	81 lb	Standard deviation per plot	10	12	CO2	PO1	7	
	Set of 121 plots	Set of 81 plots													
Mean yield per plot	84 lb	81 lb													
Standard deviation per plot	10	12													
		UNIT - IV													
5	a)	In the past a machine has produced washers having a thickness of 0.5mm. To find whether the machine is in proper working order, a sample of 10 washers is chosen for which the mean thickness is 0.53mm and the standard deviation is 0.03mm Test the hypothesis that the machine is in proper working order using a level of significance of 5%.	CO2	PO1	6										
	b)	A textile company is trying to develop a green dye which will produce a consistent shade of green in cotton material. Two different formulations are tried out, the first on 10 test squares of cotton and the second on 8 test squares. The depth of colour of each square of material is then assessed on a points scale. The standard deviation of the 10 squares on which the first dye was used is 3.0 and that of the 8 squares treated with the second dye is 6.4. Does this establish that the both dyes produce same results?	CO2	PO1	7										
	c)	The number of defects in printed circuit boards is hypothesized to follow a Poisson distribution. A random sample of 60 printed boards have been collected, and the number of defects observed is as shown below: <table><tr><td>Number of defects</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Observed frequency</td><td>32</td><td>15</td><td>9</td><td>4</td></tr></table> <p>Is there evidence that the Poisson distribution fits well to the given data?</p>	Number of defects	0	1	2	3	Observed frequency	32	15	9	4	CO2	PO1	7
Number of defects	0	1	2	3											
Observed frequency	32	15	9	4											
		(OR)													
6	a)	A sample of 6 operatives from department-A of a factory have an average performance of 96 with a standard deviation of 4. A sample of 5 from department-B have an average of 92 with a standard deviation of 3.5. Do these samples establish that the overall averages of the operatives in these two departments differ at 0.01 level of significance?	CO2	PO1	6										
	b)	The following random samples are measurements of the heat-producing capacity (in millions of calories per ton) of specimens of coal from two mines: <table><tr><td>Mine-1: 8260, 8130, 8350, 8070, 8340</td></tr><tr><td>Mine-2: 7950, 7890, 7900, 8140, 7920, 7840</td></tr></table> <p>Use the 0.02 level of significance to test whether it is reasonable to assume that the variances of the two populations sampled are equal.</p>	Mine-1: 8260, 8130, 8350, 8070, 8340	Mine-2: 7950, 7890, 7900, 8140, 7920, 7840	CO2	PO1	7								
Mine-1: 8260, 8130, 8350, 8070, 8340															
Mine-2: 7950, 7890, 7900, 8140, 7920, 7840															

	c)	Two sample polls of votes for 2 candidates A and B are taken from residents of different areas. The results are given below. Examine whether the nature of the area is related to voting preferences in this election using Chi Square distribution at 5% level of significance. <div><table><tr><td></td><td colspan="2">Votes for</td></tr><tr><td>Area</td><td>A</td><td>B</td></tr><tr><td>Rural</td><td>620</td><td>380</td></tr><tr><td>Urban</td><td>550</td><td>450</td></tr></table></div>		Votes for		Area	A	B	Rural	620	380	Urban	550	450	CO2	PO1	7																													
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Area	A	B																																												
Rural	620	380																																												
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		UNIT-V																																												
7	a)	Identify the design and then analyze the data using 1% level of significance. <div><table><tr><td>A-4</td><td>B-5</td><td>D-10</td><td>E-13</td><td>C-7</td><td>B-8</td><td>A-3</td></tr><tr><td>A-3</td><td>C-4</td><td>D-9</td><td>E-10</td><td>B-8</td><td>D-4</td><td>A-3</td></tr><tr><td>D-9</td><td>C-7</td><td>E-8</td><td>B-7</td><td>E-10</td><td>A-12</td><td>B-7</td></tr><tr><td>D-3</td><td>A-9</td><td>E-7</td><td>B-7</td><td>C-10</td><td>A-7</td><td>E-7</td></tr></table></div>	A-4	B-5	D-10	E-13	C-7	B-8	A-3	A-3	C-4	D-9	E-10	B-8	D-4	A-3	D-9	C-7	E-8	B-7	E-10	A-12	B-7	D-3	A-9	E-7	B-7	C-10	A-7	E-7	CO1	PO1	10													
A-4	B-5	D-10	E-13	C-7	B-8	A-3																																								
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D-9	C-7	E-8	B-7	E-10	A-12	B-7																																								
D-3	A-9	E-7	B-7	C-10	A-7	E-7																																								
	b)	A manufacturing firm wants to investigate the effects of 5 colour additives on the acting time of a new concrete mix. Variations in the setting times can be expected from day-to-day changes and humidity and also from the different workers who prepare the test molds. The data is given below with the letters A, B, C, D and E representing the five additives. The setting times, in hours, for the 25 molds are shown below. <div><table><tr><td rowspan="2">Worker</td><td colspan="5">Day</td></tr><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>1</td><td>D-10.7</td><td>E-10.3</td><td>B-11.2</td><td>A-10.9</td><td>C-10.5</td></tr><tr><td>2</td><td>E-11.3</td><td>C-10.5</td><td>D-12</td><td>B-11.5</td><td>A-10.3</td></tr><tr><td>3</td><td>A-11.8</td><td>B-10.9</td><td>C-10.5</td><td>D-11.3</td><td>E-7.5</td></tr><tr><td>4</td><td>B-14.1</td><td>A-11.6</td><td>E-11</td><td>C-11.7</td><td>D-11.5</td></tr><tr><td>5</td><td>C-14.5</td><td>D-11.5</td><td>A-11.5</td><td>E-12.7</td><td>B-10.9</td></tr></table></div> <div>Using appropriate design analyse the data at 5% level of significance.</div>	Worker	Day					1	2	3	4	5	1	D-10.7	E-10.3	B-11.2	A-10.9	C-10.5	2	E-11.3	C-10.5	D-12	B-11.5	A-10.3	3	A-11.8	B-10.9	C-10.5	D-11.3	E-7.5	4	B-14.1	A-11.6	E-11	C-11.7	D-11.5	5	C-14.5	D-11.5	A-11.5	E-12.7	B-10.9	CO1	PO1	10
Worker	Day																																													
	1	2	3	4	5																																									
1	D-10.7	E-10.3	B-11.2	A-10.9	C-10.5																																									
2	E-11.3	C-10.5	D-12	B-11.5	A-10.3																																									
3	A-11.8	B-10.9	C-10.5	D-11.3	E-7.5																																									
4	B-14.1	A-11.6	E-11	C-11.7	D-11.5																																									
5	C-14.5	D-11.5	A-11.5	E-12.7	B-10.9																																									
