

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

June / July 2024 Semester End Main Examinations

Programme: B.E.

Semester: VI

Branch: Institutional Elective

Duration: 3 hrs.

Course Code: 23MA60ENME

Max Marks: 100

Course: Numerical Methods for Engineers

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

UNIT - I			<i>CO</i>	<i>PO</i>	Marks												
1	a)	Find the approximate solution of the non-linear equations $x^2y + y^3 = 10$, $xy^2 - x^2 = 3$ near $x = 0.8$, $y = 2.2$ using Newton's method. Perform two iterations.	<i>CO1</i>	<i>PO1</i>	06												
	b)	Apply Thomas algorithm to find the solution of the tri-diagonal systems using $\begin{bmatrix} 2.04 & -1 & 0 & 0 \\ -1 & 2.04 & -1 & 0 \\ 0 & -1 & 2.04 & -1 \\ 0 & 0 & -1 & 2.04 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} 40.8 \\ 0.8 \\ 0.8 \\ 200.8 \end{bmatrix}$	<i>CO1</i>	<i>PO1</i>	07												
	c)	Find all eigenvalues and corresponding eigenvectors of the matrix $A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$ using Jacobi's method.	<i>CO1</i>	<i>PO1</i>	07												
UNIT - II																	
2	a)	Apply Sterling's interpolation to compute $f(0.25)$ from the following table: <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td><td>0.20</td><td>0.22</td><td>0.24</td><td>0.26</td><td>0.28</td></tr> <tr> <td>$y = f(x)$</td><td>1.6596</td><td>1.6698</td><td>1.6803</td><td>1.6912</td><td>1.7023</td></tr> </table>	x	0.20	0.22	0.24	0.26	0.28	$y = f(x)$	1.6596	1.6698	1.6803	1.6912	1.7023	<i>CO1</i>	<i>PO1</i>	06
x	0.20	0.22	0.24	0.26	0.28												
$y = f(x)$	1.6596	1.6698	1.6803	1.6912	1.7023												
	b)	Obtain the cubic spline approximation valid in $[3,4]$ for the function defined by the data: <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td><td>1</td><td>2</td><td>3</td><td>4</td></tr> <tr> <td>$f(x)$</td><td>3</td><td>10</td><td>29</td><td>65</td></tr> </table> Under the natural spline conditions $f''(1) = M(1) = 0$ and $f''(4) = M(4) = 0$. Hence find $f(3.5)$.	x	1	2	3	4	$f(x)$	3	10	29	65	<i>CO1</i>	<i>PO1</i>	07		
x	1	2	3	4													
$f(x)$	3	10	29	65													

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

	c)	Evaluate $\int_{y=0}^1 \int_{x=0}^1 \frac{\sin(xy)}{1+xy} dx dy$ using Trapezoidal rule with $h = \frac{1}{3}$ in x -direction and $k = \frac{1}{3}$ in y -direction.	CO1	PO1	07
		UNIT - III			
3	a)	Find an approximate solution of simultaneous ODEs $\frac{dy}{dx} = -2y + 4e^{-x}$, $y(0) = 2$ and $\frac{dz}{dx} = -\frac{yz^2}{3}$, $z(0) = 4$ at $x = 0.2$ with step size $h = 0.2$ using Runge-Kutta 4 th order method.	CO1	PO1	10
	b)	Apply Adams-Basforth method to compute $y(0.2)$ from the differential equation $\frac{dy}{dx} = y^2 \sin(t)$, given $y(0) = 1$, $y(0.05) = 1.00125$, $y(0.1) = 1.00502$, and $y(0.15) = 1.01136$	CO1	PO1	06
	c)	Represent the differential equation $y''' + 3y'' + y' + 3y = \sin(2x)$, $0 \leq x \leq 1$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = 2$ in the fundamental matrix form by reducing it into first order system.	CO1	PO1	04
		OR			
4	a)	Compute $y(1.4)$ by using Milne's method given $x^2 \frac{dy}{dx} + xy = 1$, subject to conditions $y(1) = 1$, $y(1.1) = 0.996$, $y(1.2) = 0.986$, and $y(1.3) = 0.972$.	CO1	PO1	06
	b)	Find an approximate solution of system of ODEs $\frac{dy}{dx} = -0.5y$, $y(0) = 4$; $\frac{dz}{dx} = 4 - 0.3z - 0.1y$; $z(0) = 6$ at $x = 1$ using Runge-Kutta 2 nd order method with step size $h = 0.5$.	CO1	PO1	08
	c)	Solve the autonomous system $\frac{dy}{dt} = -3x + 4y$ and $\frac{dx}{dt} = 3x - 5y$ using matrix method.	CO1	PO1	06
		UNIT - IV			
5	a)	Find the numerical solutions of the boundary value problem $y'' + 2y' + y = x^2$, $y(0) = 0.2$, $y(1) = 0.8$ using shooting technique along with RK2 method with step size $h = 0.5$ and initial guess is $y'(0) = \alpha = 0.5$.	CO1	PO1	10
	b)	Find the numerical solutions of the following ODE using Finite Difference method $y'' - \left(1 - \frac{x}{5}\right)y = x$, $y(1) = 2$, $y(3) = -1$, with step size 0.5.	CO1	PO1	10
		OR			

	6	a)	Solve the differential equations $y'' + \frac{4x}{1+x^2}y' + \frac{2}{1+x^2}y = 0$, in $0 \leq x \leq 2$, using cubic spline method subject to conditions $y(0) = 1, y(2) = 0.2$ with step size $h = 1$.	CO1	PO1	10
		b)	Apply the finite difference method with Trapezoidal rule to solve the integral equations $f(x) + \int_0^1 x(e^{xt} - 1)f(t)dt = e^x - x$ by taking step size $h = \frac{1}{3}$.	CO1	PO1	10
			UNIT - V			
	7	a)	Solve the boundary value problem $\nabla^2 u = -20\cos(3\pi x)\sin(2\pi y)$ on the unit square with boundary conditions $u(0, y) = y^2, u(1, y) = 1, u(x, 0) = x^3, u(x, 1) = 1$ using central difference approximation to second order partial derivatives with step size $1/3$ in both x and y directions.	CO1	PO1	10
		b)	Derive the explicit finite difference formula to solve the partial differential equation $\frac{\partial^2 u}{\partial t^2} = c \frac{\partial^2 u}{\partial x^2}$. Also find the displacement in string of length 2cm at $t = 0.2$ sec and $t = 0.4$ sec, if the initial displacement is $u(x, 0) = x$ and initial velocity is zero and the ends of string are fixed, by taking $c = 0.25$ and $h = 0.2$.	CO1	PO1	10
