U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

June 2025 Semester End Main Examinations

Programme: B.E.
Branch: Institutional Elective
Course Code: 20MA6IENME
Course: Numerical Methods for Engineers

Semester: VI
Duration: 3 hrs.
Max Marks: 100

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	CO	PO	Marks
1	a)	Prove that the rate of convergence of Newton's method is second-order.	1	1	6
	b)	Apply Jacobi's method to approximate all eigenvalues and the corresponding eigenvectors of the matrix $A=\begin{bmatrix}1 & -2 & 4\\ -2 & 5 & -2\\ 4 & -2 & 1\end{bmatrix}$.	1	1	7
	c)	Solve non-linear system of equations $x^2+xy-10=0$; $y+3xy^2-57=0$ near $x=1.5; y=3.5$ using the fixed-point iteration method. Carryout two iterations.	1	1	7
		OR			
2	a)	Apply Given's method to reduce the following matrix into tridiagonal form $\begin{bmatrix}1 & 1/2 & 1/3\\ 1/2 & 1/3 & 1/4\\ 1/3 & 1/4 & 1/5\end{bmatrix}$.	1	1	6
	b)	Apply Thomas algorithm to approximate the solution of the tridiagonal system $\begin{bmatrix}4 & 3 & 0 & 0\\ 2 & 2 & 3 & 0\\ 0 & 2 & 1 & 2\\ 0 & 0 & 2 & 5\end{bmatrix}\begin{bmatrix}T_1\\ T_2\\ T_3\\ T_4\end{bmatrix}=\begin{bmatrix}10\\ 2\\ -3\\ 5\end{bmatrix}$.	1	1	7
	c)	Apply Newton's method to find an approximate solution of the system of equations $x^2+3x+y=5$ and $x^2+3y^2=4$ near $x=0.5$ and $y=0.5$. Perform two iterations.	1	1	7

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<tr><td></td><td></td><td></td><td>UNIT - 2</td><td></td><td></td><td></td></tr>
<tr><td></td><td>3</td><td>a)</td><td>Approximate the derivative of the function $f(x)=e^{x}\sin(x)$ at the point $x=1$ using the central finite difference formula and then refine the results using Richardson extrapolation starting with $h = 0.5, 0.25$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find the volume under the surface $f(x,y)=e^{(x^2+y^2)}$ by the evaluating the double integral $\int_0^1\int_0^1 f(x,y)dxdy$ using Trapezoidal rule by taking three equal number of subintervals in both x and y directions.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Obtain the cubic spline interpolation in [2,3] for $f(x)$ from the data with $M(0) = 0, M(4) = 0$ and estimate $f(2.5)$
<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$f(x)$</td><td>3</td><td>10</td><td>29</td><td>65</td></tr></table></td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>4</td><td>a)</td><td>Obtain the piecewise linear interpolating polynomials for the function $f(x)$ using the given data and find $f(3)$ and $f(7)$.
<table><tr><td>x</td><td>1</td><td>2</td><td>4</td><td>8</td></tr><tr><td>f(x)</td><td>3</td><td>7</td><td>21</td><td>73</td></tr></table></td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Apply Romberg's integration method to compute $\int_4^{5.2}\log x\,dx$ with Simpson's rule by taking step size $h=0.3$ and 0.6.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Evaluate $\int_{y=1}^{1.5}\int_{x=1}^{2}\frac{dxdy}{x+y}$ using Simpson's rule with $h=0.5$ along x-axis and $k=0.25$ along y-axis.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT – 3</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Derive the i). forward finite difference, backward finite difference and central finite difference approximations of $\frac{dy}{dx}$ and ii) central finite difference approximation of $\frac{d^2y}{dx^2}$.</td><td>1</td><td>1</td><td>5</td></tr>
<tr><td></td><td></td><td>b)</td><td>Apply Adams-Bashforth method to approximate the solutions of the ODE $y'=1+y^2$ at $x=0.8$ subject to the conditions $y(0)=0$, $y(0.2)=0.2027$, $y(0.4)=0.4228; y(0.6)=0.6841$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Apply Runge-Kutta fourth order method to approximate the solution of the simultaneous differential equations $y'=2y+u, y(0)=1;$ $u'=3y+4u, u(0)=1$ at $x=0.2$ with step size $h = 0.2$.</td><td>1</td><td>1</td><td>8</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
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	6	a)	Reduce the differential equation $u''' + 2u'' + u' - u = \cos(t)$, $0 \le t \le 1$; $u(0) = 0$, $u'(0) = 1, u''(0) = 2$ in to first order initial value problems.	1	1	**5**
		b)	Apply Milne's method to approximate the solution of $x^2 \frac{dy}{dx} + xy = 1$ at $x = 1.400$ given $y(1.0) = 1.000$, $y(1.1) = 0.996$, $y(1.2) = 0.986$ and $y(1.3) = 0.972$.	1	1	**5**
		c)	Apply Runge-Kutta second order method to approximate the solution of simultaneous differential equations, $y' = u$, $y(0) = 1$; $u' = -4y - 2u, u(0) = 1$ at $x = 0.2$ with step size $h = 0.1$.	1	1	**10**
			UNIT - 4			
	7	a)	Apply finite difference method to solve $u'' = u - 4xe^x$, in $0 \le x \le 1$, subject to the conditions $u(0) - u'(0) = -1$, and $u(1) + u'(1) = -e$ with $h = \frac{1}{3}$.	1	1	**10**
		b)	Find the numerical solutions of the boundary value problem $y'' + 2y' + y = x^2$, $y(0) = 0.2, y(1) = 0.8$ applying shooting method with step size $h = 0.5$ by taking initial guess $y'(0) = \alpha = 0.5$. Obtain its first correction.	1	1	**10**
			OR			
	8	a)	Apply cubic spline method to solve the boundary value problem $x^2 y'' + xy' - y = 0$; $y(1) = 1$ and $y(2) = 0.5$, with step size $h = 0.5$.	1	1	**10**
		b)	Apply finite difference method to solve the integral equations $f(x) = \frac{15x - 2}{18} + \frac{1}{3}\int_0^1 (x + t)\, f(t)dt$ using Trapezoidal's rule by taking step size $h = 0.5$.	1	1	**10**
			UNIT - 5			
	9	a)	Solve $u_{xx} + u_{yy} = 0$ for the region bounded by $1 \le x \le 2$, $0 \le y \le 1$, subject to the boundary conditions $u(1, y) = \ln(y^2 + 1)$; $u(x, 0) = 2\ln(x)$; $u(2, y) = \ln(y^2 + 4)$ and $u(x, 1) = \ln(x^2 + 1)$ with $h = k = 1/3$.	1	1	**10**
		b)	Solve Poisson's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -10(x^2 + y^2 + 10)$ for the region bounded by $0 \le x, y \le 3$, subject to the conditions $u(0, y) = u(x, 0) = u(3, y) = u(x, 3) = 0$ with $h = k = 1$.	1	1	**10**
			OR			

	10	a)	Solve $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=xe^y, 0<x<2,\ 0<y<1$ subject to the boundary conditions $u(x,0)=x$, $u(0,y)=0$, $u(2,y)=2e^y$ and $u(x,1)=xe$ with step size $\frac{2}{3}$ and $\frac{1}{3}$ along x and y directions respectively.	*1*	*1*	**10**
		b)	Solve $\nabla^2 u=0$ over $R:\{(0,1)\times(0,1)\}$ with $h=k=\frac{1}{3}$ subject to the conditions $u(x,1)=1$, $u(1,y)=1, u(x,0)=0$ and $u(0,y)=0$.	*1*	*1*	**10**

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