U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

June 2025 Semester End Main Examinations

Program	**: B. E**	**Semester**	**: VI**
Branch	**: Institutional Elective**	**Duration**	**: 03 Hrs.**
Course Code	**: 23MA6OENME**	**Max. Marks**	**: 100**
Course Name	**: Numerical Methods for Engineers**		

Instructions: 1. Each unit has an internal choice; answer one complete question from each unit.
2. Missing data, if any, may be suitably assumed

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

			CO	PO	Marks
		UNIT - 1			
1	a)	Prove that the rate of convergence of Newton's method is second order.	1	1	**6**
	b)	Approximate all eigenvalues and corresponding eigenvectors of the matrix $A = \begin{bmatrix} 1 & \sqrt{2} & 4 \\ \sqrt{2} & 3 & \sqrt{2} \\ 4 & \sqrt{2} & 1 \end{bmatrix}$ using Jacobi's method.	1	1	**7**
	c)	Solve non-linear equations $x^2 + xy - 10 = 0;\ y + 3xy^2 - 57 = 0;$ near $x = 1.5; y = 3.5$ using fixed point iteration method. Perform two iterations.	1	1	**7**
		OR			
2	a)	Apply Given's method to reduce the matrix $\begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{bmatrix}$ into tri-diagonal form.	1	1	**6**
	b)	Solve the tri-diagonal system $\begin{bmatrix} 1 & -2 & & & \\ 2 & 4 & 5 & & \\ & 8 & -9 & 2 & \\ & & 6 & 3 & 4 \\ & & & 6 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \\ 4 \\ 5 \end{bmatrix}$ using the Thomas algorithm.	1	1	**7**
	c)	Apply Newton Raphson method to approximate the real root of $\ln\left(x^2 + y\right) - 1 + y = 0, \sqrt{x} + xy = 0$ near $x = 2.4, y = -0.6$. Perform two iterations.	1	1	**7**
		UNIT - 2			
3	a)	Obtain the piecewise linear interpolation for $f(x)$ from the following data and estimate $f(3)$ and $f(1.5)$. x: 1, 2, 4, 8 $f(x)$: 3, 7, 21, 73	1	1	**6**

x	1	2	4	8
$f(x)$	3	7	21	73

<table>
<tr><td></td><td></td><td>b)</td><td>Apply Romberg’s integration method along with Trapezoidal rule to compute $\int_0^1 \left(1+\frac{\sin x}{x}\right)dx$ with $h = 0.25, 0.125$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Apply Sterling’s central difference interpolation formula to compute y at $x = 0.25$ from the following data:
<table><tr><td>x</td><td>0.20</td><td>0.22</td><td>0.24</td><td>0.26</td><td>0.28</td><td>0.30</td></tr><tr><td>$f(x)$</td><td>1.6596</td><td>1.669</td><td>1.6803</td><td>1.6912</td><td>1.7023</td><td>1.7138</td></tr></table></td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>4</td><td>a)</td><td>Approximate the derivative of the function $f(x) = e^x \sin(x)$ at the point $x = 1$ using the centered difference formula and then refine the results using Richardson extrapolation starting with h = 0.5, 0.25, 0.125.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find the volume under the surface $f(x,y) = e^{(x^2+y^2)}$ by evaluating the double integral $\int_0^1\int_0^1 f(x,y)dxdy$ using Trapezoidal rule by taking three equal subintervals in both x and y directions.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Compute $f(1.5)$ and $f(2.5)$ using cubic spline interpolation from the data given below with $M(1) = 0$ and $M(4) = 0$.
<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$f(x)$</td><td>3</td><td>10</td><td>29</td><td>65</td></tr></table></td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 3</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Verify the existence and uniqueness of the solution of the initial value problem $y' = y^{1/3}, y(0) = 0$ over the domain
i) $0 \le t \le 2,\ 0 \le y \le 1$, (ii) $0 \le t \le 2,\ c \le y \le d,\ 0 < c < d$. If so, find the Lipschitz constant for each case.</td><td>1</td><td>1</td><td>5</td></tr>
<tr><td></td><td></td><td>b)</td><td>The differential equations that govern the prey and predator population are given by
$\frac{dy(t)}{dt} = 3y(t) + 2z(t); \frac{dz(t)}{dt} = 4y(t) + z(t); y(0) = 1, z(0) = 1$.
Applying matrix method find the prey $x(t)$ and predator $y(t)$ population.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>c)</td><td>Apply RK4 to solve $y' = 2y + u, y(0) = 1; u' = 3y + 4u, u(0) = 1$ at $x = 0.2$ with step size $h = 0.2$.</td><td>1</td><td>1</td><td>9</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>6</td><td>a)</td><td>Derive the forward difference, backward difference and central difference of $\frac{dy}{dx}$ and central difference approximation of $\frac{d^2y}{dx^2}$.</td><td>1</td><td>1</td><td>4</td></tr>
<tr><td></td><td></td><td>b)</td><td>The differential equations that govern the prey and predator population are given by
$\frac{dx(t)}{dt} = -2x(t) + 3y(t); \frac{dy(t)}{dt} = -x(t) + 2y(t); x(2) = 2, y(2) = 4$.
Applying matrix method find the prey $x(t)$ and predator $y(t)$ population.</td><td>1</td><td>1</td><td>6</td></tr>
</table>

		c)	Using RK2, approximate the solution of the system of the initial value problems $\frac{dy}{dx}=-2y+4e^{-x}, y(0)=2; \frac{dz}{dx}=-\frac{yz^2}{3}, z(0)=4$ with step size $h = 0.1$ at $x = 0.2$.	1	1	**10**
			UNIT - 4			
	7	a)	Apply cubic spline method to solve the boundary value problem $x^2y''+xy'-y=0;\ y(1)=1$ and $y(2)=0.5$, with step size $h=0.5$	1	1	**10**
		b)	Apply finite difference method to solve the boundary value problem $u''+\frac{4x}{1+x^2}u'+\frac{2}{1+x^2}u=0$, in $0\le x\le 1$, subject to the conditions $u(0)=1, u(1)=\frac{1}{2}$ with $h=\frac{1}{3}$.	1	1	**10**
			OR			
	8	a)	Find the numerical solutions of the boundary value problem $y''+2y'+y=x^2,\quad y(0)=0.2, y(1)=0.8$ using shooting technique with step size $h=0.5$ with initial guess $y'(0)=\alpha=0.5$. Obtain its first correction.	1	1	**10**
		b)	Apply finite difference method along with Trapezoidal rule to solve the integral equations $f(x)=\frac{15x-2}{18}+\frac{1}{3}\int_0^1 (x+t)\, f(t)dt$ taking step size $h=\frac{1}{3}$.	1	1	**10**
			UNIT - 5			
	9	a)	Compute the numerical solution of one-dimensional heat equation $u_t=u_{xx}$ with $u(t,0)=0=u(t,1)$ and $u(0,x)=\sin(\pi x)$ up to five time levels with step size $h = 0.2$ along the x-directions.	1	1	**10**
		b)	Solve Poisson's equation $\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=-10(x^2+y^2+10)$ for the region bounded by $0\le x,y\le 3$, subject to the conditions $u(0,y)=u(x,0)=u(3,y)=u(x,3)=0$ with h=k=1.	1	1	**10**
			OR			
	10	a)	Solve $\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=0$ for the region bounded by $0\le x\le 1, 0\le y\le 1$ with $h = k = 1/3$ subject to the boundary conditions $u(0,y)=u(x,0)=u(1,y)=u(x,1)=0$.	1	1	**10**
		b)	Solve the wave equation $u_{tt}=4u_{xx}$ taking $h = 1$ and $k = 1/2$ up to $t = 2$ with the boundary conditions $u(0,t)=u(4,t)=0; u_t(x,0)=0$ and $u(x,0)=x(4-x)$.	1	1	**10**

******